pst-euclide

A PSTricks package for drawing geometric pictures; v.1.75

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The `pst-eucl` package allow the drawing of Euclidean geometric figures using LaTeX macros for specifying mathematical constraints. It is thus possible to build point using common transformations or intersections. The use of coordinates is limited to points which controlled the figure.

I would like to thanks the following persons for the help they gave me for development of this package:

- Denis Girou pour ses critiques pertinentes et ses encouragement lors de la découverte de l'embryon initial et pour sa relecture du présent manuel;
- Michael Vulis for his fast testing of the documentation using VTEX which leads to the correction of a bug in the `PostScript` code;
- Manuel Luque and Olivier Reboux for their remarks and their examples.
- Alain Delplanque for its modification theorems on automatic placing of points name and the ability of giving a list of points in `\pstGeonode`.

Thanks to: Manuel Luque; Thomas Söll.

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Part I. The package

1. Special specifications

1.1. PSTricks Options

The package activates the `\SpecialCoor` mode. This mode extend the coordinates specification. Furthermore the plotting type is set to `dimen=middle` , which indicates that the position of the drawing is done according to the middle of the line. Please look at the user manual for more information about these setting.

At last, the working axes are supposed to be (ortho)normed.

1.2. Conventions

For this manual, I used the geometric French conventions for naming the points:

- O is a centre (circle, axes, symmetry, homothety, rotation);
- I defined the unity of the abscissa axe, or a midpoint;
- J defined the unity of the ordinate axe;
- A, B, C, D are points ;
- M' is the image of M by a transformation ;

At last, although these are nodes in PSTricks, I treat them intentionally as points.

2. Basic Objects

2.1. Points

`\pstGeonode [Options]` $(x_1,y_1)\{A_1\}(x_2,y_2)\{A_1\}\ldots(x,y)\{A_n\}$

This command defines one or more geometrical points associated with a node in the default cartesian coordinate system. Each point has a node name A_i which defines the default label put on the picture. This label is managed by default in mathematical mode, the boolean parameter `PtNameMath` (default `true`) can modify this behavior and let manage the label in normal mode. It is placed at a distance of `PointNameSep` (default 1em) of the center of the node with a angle of `PosAngle` (default 0). It is possible to specify another label using the parameter `PointName=default` , and an empty label can be specified by selecting the value `none`, in that case the point will have no name on the picture.

The point symbol is given by the parameter `PointSymbol=*` . The symbol is the same as used by the macro `\psdot`. This parameter can be set to `none`, which means that the point will not be drawn on the picture.

Here are the possible values for this parameter:

- `*`: ●
- `o`: ○
- `+`: +
- `x`: ·
- `asterisk` : ⋆
- `oplus`: ⁻
- `otimes`: ˜
- `triangle`: △
- `triangle*`: ▲
- `square`: □
- `square*`: ■
- `diamond`: ◊
- `diamond*`: ⧫
- `pentagon`: ⬠
- `pentagon*`: ⬟
- `|`: |

Furthermore, these symbols can be controlled with some others PSTricks, several of these are :

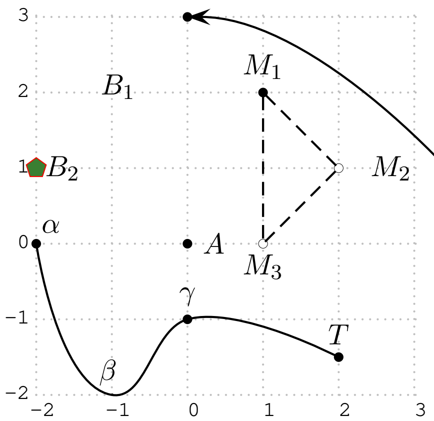
- their scale with `dotscale`, the value of whom is either two numbers defining the horizontal and vertical scale factor, or one single value being the same for both,
- their angle with parameter `dotangle`.

Please consult the PSTricks documentation for further details. The parameters `PosAngle`, `PointSymbol`, `PointName` and `PointNameSep` can be set to :

- either a single value, the same for all points ;
- or a list of values delimited by accolads `{ ... }` and separated with comma *without any blanks*, allowing to differenciate the value for each point.

In the later case, the list can have less values than point which means that the last value is used for all the remaining points. At least, the parameter setting `CurveType=none` can be used to draw a line between the points:

- opened `polyline` ;
- closed `polygon` ;
- open and curved `curve`.




```
\begin{pspicture}[showgrid=true](-2,-2)(3,3)
\pstGeonode{A}
\pstGeonode[PosAngle=-135, PointNameSep=1.3](0,3){B_1}
\pstGeonode[PointSymbol=pentagon, dotscale=2, fillstyle=solid,
        fillcolor=OliveGreen, PtNameMath=false,
        PointName=$B_2$, linecolor=red](-2,1){B2}
\pstGeonode[PosAngle={90,0,-90}, PointSymbol={*,o},
        linestyle=dashed, CurveType=polygon,
        PointNameSep={1em,2em,3mm}]
  (1,2){M_1}(2,1){M_2}(1,0){M_3}
\pstGeonode[PosAngle={50,100,90}, PointSymbol={*,x,*},
        PointNameSep=3mm, CurveType=curve,
        PointName={\alpha,\beta,\gamma,default}]
  (-2,0){alpha}(-1,-2){beta}(0,-1){gamma}(2,-1.5){T}
\end{pspicture}
```

Obviously, the nodes appearing in the picture can be used as normal PSTricks nodes. Thus, it is possible to reference a point from here.

After v1.65, we add macros `\pstAbscissa` and `\pstOrdinate` to get the abscissa and ordinate of the specified node, so it is possible to define a new node from an already constructed node with them.

```
\pstAbscissa{A}
\pstOrdinate{A}
```

Note that the value of abscissa or ordinate are transformed to the `User coordinate`, and then put into the stack of `PostScript`, so they can be used to do some compound arithmetic without concerned the `xunit` and `yunit` in the `PSTricks SpecialCoor` function. You need the other third package to do float arithmetic operation, like `\pscalculate` [1] to generate the numerical values, or the expandable command `\fpeval`[2] to get a purely numerical result.

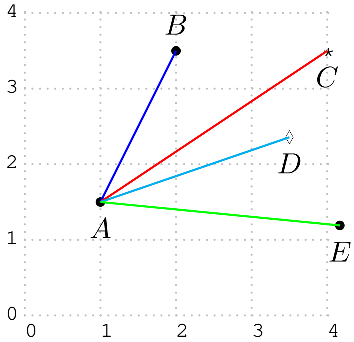
The macro `\pstMoveNode` use them to move node A by abscissa increment dx and ordinate increment dy to get the target node B.

1 Provided by package `pst-calculate`, sometimes it results the numbers more than 9 fraction digits, which are not supported good by PSTricks with '! number too big' issue.

2 Provided by package `xfp`, it can truncate the fraction part digits using the `trunc` function perfectly, e.g. `\fpeval{trunc(18/7,3)}`.

\pstMoveNode [Options] $(dx,dy)\{A\}\{B\}$

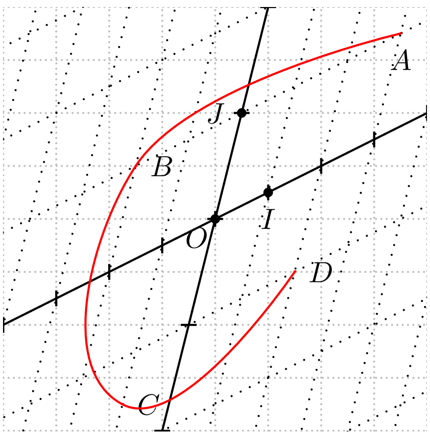
for example:




```
\begin{pspicture}[showgrid=true](0,0)(4,4)
\def\ra{3.0}\def\rb{4.0}
\pstGeonode[PosAngle=-90](1.0,1.5){A}
\pstGeonode[PosAngle=90](! \pstAbscissa{A} 1 add \pstOrdinate{A}
    2 add){B}
\pstLineAB[linecolor=blue]{A}{B}
\pstMoveNode[PosAngle=-90,PointSymbol=asterisk](3,2){A}{C}
\pstLineAB[linecolor=red]{A}{C}
\pstMoveNode[PosAngle=-90,PointSymbol=diamond](\pscalculate{sqrt
    (\ra*\ra+\rb*\rb)/2},\pscalculate{\ra*\rb/(2*(\ra+\rb))}){A
    }{D}
\pstLineAB[linecolor=cyan]{A}{D}
\pstMoveNode[PosAngle=-90](\pstAbscissa{B} 3 div,\pstOrdinate{B}
    neg 3 div){D}{E}
\pstLineAB[linecolor=green]{A}{E}
\end{pspicture}
```

\pstOIJGeonode creates a list of points in the landmark $(O;I;J)$. Possible parameters are PointName, PointNameSep, PosAngle, PointSymbol, and PtNameMath.

\pstOIJGeonode [Options] $(x_1,y_1)\{A_1\}\{O\}\{I\}\{J\}\ (x_2,y_2)\{A_2\}\ldots(x,y)\{A_n\}$




```
\psset{unit=.7}
\begin{pspicture*}[showgrid=true](-4,-4)(4,4)
  \pstGeonode[PosAngle={-135,-90,180}]{O}(1,0.5){I}(0.5,2){J}
  \pstLineAB[nodesep=10]{O}{I}
  \pstLineAB[nodesep=10]{O}{J}
  \multips(-5,-2.5)(1,0.5){11}{\psline(0,-.15)(0,.15)}
  \multips(-2,-8)(0.5,2){9}{\psline(-.15,0)(.15,0)}
  \psset{linestyle=dotted}%
  \multips(-5,-2.5)(1,0.5){11}{\psline(-10,-40)(10,40)}
  \multips(-2,-8)(0.5,2){9}{\psline(-10,-5)(10,5)}
  \psset{PointSymbol=x, linestyle=solid}
  \pstOIJGeonode[PosAngle={-90,0}, CurveType=curve,
   linecolor=red] (3,1){A}{O}{I}{J}(-2,1){B}(-1,-1.5){C}(2,-1){D}
\end{pspicture*}
```

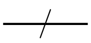
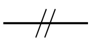
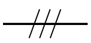
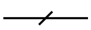
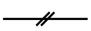
2.2. Segment mark

A segment can be drawn using the \ncline command. However, for marking a segment there is the following command:

\pstSegmentMark [Options] $\{A\}\{B\}$

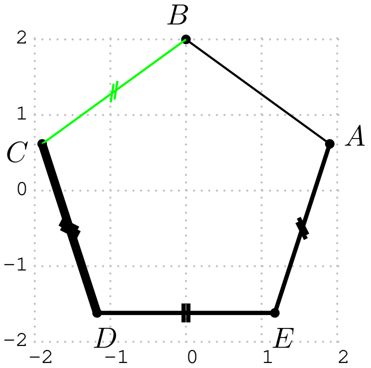
The symbol drawn on the segment is given by the parameter SegmentSymbol. Its value can be any valid command which can be used in math mode. Its default value is MarkHashh, which produced two slashes on the segment. The segment is drawn.

Several commands are predefined for marking the segment:

- pstslash 
- pstslashh 
- pstslashhh 
- MarkHash 
- MarkHashh 
- MarkHashhh 

- MarkCros
- MarkCross
- MarkArrow
- MarkAroww
- MarkArrowww

The three commands of the family MarkHash draw a line whose inclination is controled by the parameter MarkAngle (default is 45). Their width and colour depends of the width and color of the line when the drawing is done, as shown is the next example.




```
\begin{pspicture}[showgrid=true](-2,-2)(2,2)
\rput{18}{%
  \pstGeonode[PosAngle={0,90,180,-90}](2,0){A}(2;72){B}
    (2;144){C}(2;216){D}(2;288){E}}
\pstSegmentMark[SegmentSymbol=none]{A}{B}
\pstSegmentMark[linecolor=green]{B}{C}
\psset{linewidth=2\pslinewidth}
\pstSegmentMark[linewidth=2\pslinewidth]{C}{D}
\pstSegmentMark[MarkAngle=90]{D}{E}
\pstSegmentMark{E}{A}
\end{pspicture}
```

The length and the separation of multiple hases can be set by MarkHashLength and MarkHashSep.

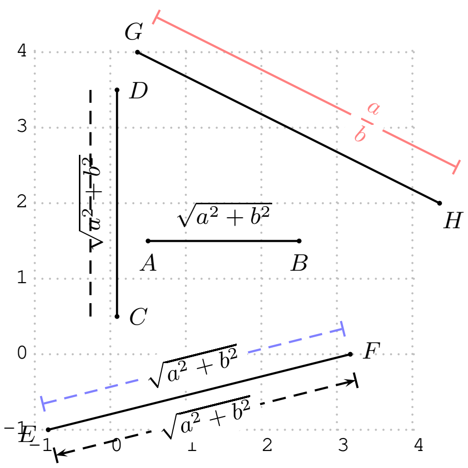
2.3. Segment labels

According to the manual of PSTricks, you can use the macros \naput, \ncput and \nbput to put the label *above*, *cover*, *below* the segment. The macro \pstLabelAB just use them to draw a ruler bar and put the label on the ruler bar.

\pstLabelAB [*] [Options] {*A*}{*B*}{*label*}

You can use the parameters of \ncline to control the ruler bar, such as linestyle, linecolor, linewidth, arrows, nodesep etc; and use the parameters of \ncput to control the label position, such as nrot, npos etc; there is another parameter offset to control the separation between the rule bar and the segment.

It does not display the ruler bar as default, and you need to setup linestyle to display it. The star version uses also the star version of the put macro (white background).




```
\begin{pspicture}[showgrid=true](-1,-1)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\pstGeonode[PosAngle=-90](0.5,1.5){A}
\pstGeonode[PosAngle=-90](2.5,1.5){B}\pstLineAB{A}{B}
\pstLabelAB{A}{B}{$\sqrt{a^2+b^2}$}
\pstGeonode[PosAngle=0](0,0.5){C}
\pstGeonode[PosAngle=0](0,3.5){D}\pstLineAB{C}{D}
\pstLabelAB[linestyle=dashed]{C}{D}{$\sqrt{a^2+b^2}$}
\pstGeonode[PosAngle=190](-1,-1){E}
\pstGeonode[PosAngle=10](3,0){F}\pstLineAB{E}{F}
\pstLabelAB*[linestyle=dashed,arrows=|-|,offset=10pt,linecolor=
    blue!50]{E}{F}{$\sqrt{a^2+b^2}$}
\pstLabelAB*[linestyle=dashed,arrows=|<->|,offset=10pt,nrot=:D]{F
    }{E}{$\sqrt{a^2+b^2}$}
\pstGeonode[PosAngle=100](0,4){G}
\pstGeonode[PosAngle=-50](4,2){H}\pstLineAB{G}{H}
\pstLabelAB*[linestyle=solid,linecolor=red!50,arrows=|-|,offset
    =15pt,nrot=:U,npos=0.7]{G}{H}{\textcolor{red!50}{$\dfrac{a}{
    b}$}}
\end{pspicture}
```

2.4. Triangles

The more classical figure, it has its own macro `\pstTriangle` for a quick definition:

`\pstTriangle [Options]` (x_1,y_1){A}(x_2,y_2){B}(x_3,y_3){C}

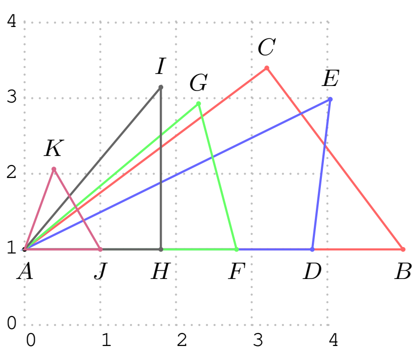
Valid optional arguments are `PointName`, `PointNameSep`, `PointSymbol`, `PointNameA`, `PosAngleA`, `PointSymbolA`, `PointNameB`, `PosAngleB`, `PointSymbolB`, `PointNameC`, `PosAngleC`, and `PointSymbolC`. In order to accurately put the name of the points, there are three parameters `PosAngleA`, `PosAngleB` and `PosAngleC`, which are associated respectively to the nodes $\langle A\rangle$, $\langle B\rangle$ and $\langle C\rangle$. Obviously they have the same meaning as the parameter `PosAngle`. If one or more of such parameters is omitted, the value of `PosAngle` is taken. If no angle is specified, points name are placed on the bissector line.

In the same way there are parameters for controlling the symbol used for each points: `PointSymbolA`, `PointSymbolB` and `PointSymbolC`. They are equivalent to the parameter `PointSymbol`. The management of the default value followed the same rule.

The macros `\pstTriangleSSS`, `\pstTriangleSAS`, `\pstTriangleAAS` and `\pstTriangleASA` are used to draw the triangle according the specified sides or angles.

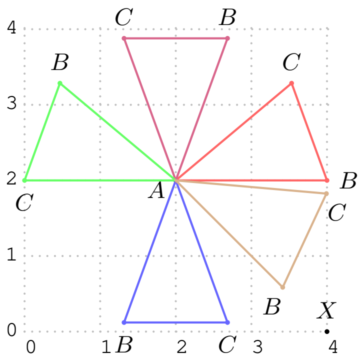
`\pstTriangleSSS [Options]` (*pos*){*A*}(*a,b,c*){*B*}{*C*}
`\pstTriangleSAS [Options]` (*pos*){*A*}(*b,∠A,c*){*B*}{*C*}
`\pstTriangleAAS [Options]` (*pos*){*A*}(*∠C,∠A,c*){*B*}{*C*}
`\pstTriangleASA [Options]` (*pos*){*A*}(*∠A,c,∠B*){*B*}{*C*}

- Macro `\pstTriangleSSS` create a triangle ABC with given $A(x_1,y_1)$, and the three sides a,b,c, it output $B(x_2,y_2)$ and $C(x_3,y_3)$.
- Macro `\pstTriangleSAS` create a triangle ABC with given $A(x_1,y_1)$, the angle of $\angle A$, and the other two sides b,c, it output $B(x_2,y_2)$ and $C(x_3,y_3)$.
- Macro `\pstTriangleAAS` create a triangle ABC with given $A(x_1,y_1)$, the angle of $\angle C$, the angle of $\angle A$, and the side of $AB=c$, it output $B(x_2,y_2)$ and $C(x_3,y_3)$.
- Macro `\pstTriangleASA` create a triangle ABC with given $A(x_1,y_1)$, the angle of $\angle A$, the angle of $\angle B$, and the side of $AB=c$, it output $B(x_2,y_2)$ and $C(x_3,y_3)$.




```
\begin{pspicture}[showgrid=true](0,0)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\pstGeonode[PosAngle=-90](0,1){A}
\pstTriangleSSS[linecolor=red!60,PosAngle={-90,90}]{A}(3,4,5){B}{
    C}
\pstTriangleSSS[linecolor=blue!60,PosAngle={-90,90}]{A
    }(2,4.5,3.8){D}{E}
\pstTriangleSAS[linecolor=green!60,PosAngle={-90,90}]{A
    }(3,40,2.8){F}{G}
\pstTriangleAAS[linecolor=black!60,PosAngle={-90,90}]{A
    }(40,50,1.8){H}{I}
\pstTriangleASA[linecolor=purple!60,PosAngle={-90,90}]{A
    }(70,1.0,60){J}{K}
\end{pspicture}
```

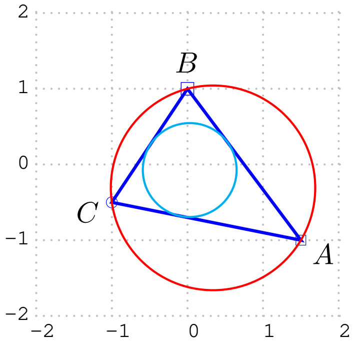
The optional parameter `pos` setup the position of the first node A, it should be ':L' for left, ':R' for right, ':U' for up and ':D' for down. You also can specify one another node X to draw a triangle base on the line AX. If you don't input this parameter, the default value is ':L'. The following example explains how to draw an isoceles triangle with the given isoceles sides and the vertex angle.




```
\begin{pspicture}[showgrid=true](0,0)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\pstGeonode[PosAngle={205,90}](2,2){A}(4,0){X}
\pstTriangleSAS[linecolor=red!60,PosAngle={0,90}]{A}(2,40,2){B}{C
    }
\pstTriangleSAS[linecolor=blue!60,PosAngle={-90,-90}](:U){A
    }(2,40,2){B}{C}
\pstTriangleSAS[linecolor=purple!60,PosAngle={90,90}](:D){A
    }(2,40,2){B}{C}
\pstTriangleSAS[linecolor=green!60,PosAngle={90,-90}](:R){A
    }(2,40,2){B}{C}
\pstTriangleSAS[linecolor=brown!60,PosAngle={-120,-60}](X){A
    }(2,40,2){B}{C}
\end{pspicture}
```

The macros \pstTriangleIC and \pstTriangleOC are used to draw the inner circle and outer circle of triangle ABC.

\pstTriangleIC [Options] $\{A\}\{B\}\{C\}$ [I] [H]
\pstTriangleOC [Options] $\{A\}\{B\}\{C\}$ [O]




```
\begin{pspicture}[showgrid](-2,-2)(2,2)
\pstTriangle[PointSymbol=square,PointSymbolC=o,
 linecolor=blue,linewidth=1.5\pslinewidth]
 (1.5,-1){A}(0,1){B}(-1,-.5){C}
\pstTriangleIC[linecolor=cyan]{A}{B}{C}
\pstTriangleOC[linecolor=red]{A}{B}{C}
\end{pspicture}
```

The center of the inner circle is called IC_O as default and the outer circle OC_O as default, but you can change the node names by the optional parameters [I] [H] and [O]. The optional node name H is a node on the inner circle, so you can operate the inner circle by center I and node H later.

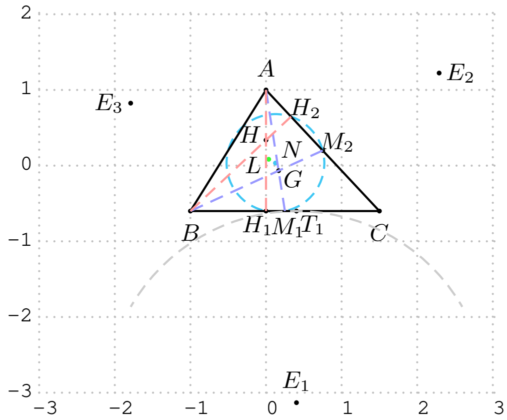
The inner center I, node H and outer circle center O are not printed out as default, but you can setup PointSymbol and PointName to display them. For example:

```
\pstTriangleIC[PosAngle={-90,160},PointName={I,none},PointSymbol={*,none}]{A}{B}{C}[I][D]
\pstTriangleIC[PosAngle=-90,PointName=I,PointSymbol=*]{A}{B}{C}[I]
\pstTriangleOC[PosAngle=90,PointSymbol=*,PointName=X]{A}{B}{C}[X]
```

The macros \pstTriangleGC, \pstTriangleHC, \pstTriangleEC, \pstTriangleNC, \pstTriangleLC are used to draw the barycenter G, the orthocentre H, the escenter E, the nine points circle center and the Lemonie point (or symmedian point) of the triangle ABC.

\pstTriangleGC [Options] $\{A\}\{B\}\{C\}\{G\}$ $[M_1]$ $[M_2]$
\pstTriangleHC [Options] $\{A\}\{B\}\{C\}\{H\}$ $[H_1]$ $[H_2]$
\pstTriangleEC [Options] $\{A\}\{B\}\{C\}\{E\}$ $[T_1]$
\pstTriangleNC [Options] $\{A\}\{B\}\{C\}\{N\}$ $[M_1]$ $[M_2]$ $[M_3]$
\pstTriangleLC [Options] $\{A\}\{B\}\{C\}\{L\}$ $[S_1]$ $[S_2]$ $[S_3]$

You can use the options of node like as PointName=..., PosAngle=..., PointSymbol=... to control the output nodes G, H, E. But if you give the optional output parameters M_1, M_2, or H_1, H_2 or T_1, then you should pass the option value in list like as PointName={...}, PosAngle={...}, PointSymbol={...}. For example,




```
\begin{pspicture}[showgrid=true](-3,-3)(3,2)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\pstGeonode[PosAngle=90](0,1){A}
\pstGeonode[PosAngle=-90](-1,-0.6){B}
\pstGeonode[PosAngle=-90](1.5,-0.6){C}
\pstTriangleGC[PointSymbol={*,none,*},PosAngle={-30,-80,30},
    PointNameSep=0.22cm]{A}{B}{C}{G}[M_1][M_2]
\pstTriangleHC[PointSymbol={*,*,none},PosAngle={160,-120,30},
    PointNameSep=0.22cm]{A}{B}{C}{H}[H_1][H_2]
\pstTriangleEC[PointSymbol={*,*},PosAngle={90,-40}]{A}{B}{C}{E
    _1}[T_1]
\pstTriangleEC[PointSymbol=*,PosAngle=0]{B}{C}{A}{E_2}
\pstTriangleEC[PointSymbol=*,PosAngle=180]{C}{A}{B}{E_3}
\pstTriangleNC[PointSymbol=*,PosAngle=40,linestyle=dashed,
    linecolor=cyan!60]{A}{B}{C}{N}
\pstTriangleLC[PointSymbol=*,PosAngle=200,linecolor=green!80,
    PointNameSep=0.22cm]{A}{B}{C}{L}
\pstLineAB{A}{B}\pstLineAB{B}{C}\pstLineAB{C}{A}
\pstCircleOA[linestyle=dashed,linecolor=gray!40]{E_1}{T
    _1}[30][150]
\pstLineAB[linestyle=dashed,linecolor=blue!40]{A}{M_1}
\pstLineAB[linestyle=dashed,linecolor=blue!40]{B}{M_2}
\pstLineAB[linestyle=dashed,linecolor=red!40]{A}{H_1}
\pstLineAB[linestyle=dashed,linecolor=red!40]{B}{H_2}
\end{pspicture}
```

2.5. Angles

Each angle is defined with three points. The vertex is the second point. Their order is important because it is assumed that the angle is specified in the direct order. The first command is the marking of a right angle:

`\pstRightAngle [Options] {A}{B}{C}`

The valid optional arguments controlling this command, excepting the ones which controlled the line, are `RightAngleType`, `RightAngleSize`, `RightAngleSize`, and `RightAngleDotDistance`.

The symbol is controlled by the parameter `RightAngleType default`. Its possible values are:

- $*$: standard symbol ;
- `german` : german symbol (given by U. Dirr) ;
- `suisseromand` : swiss romand symbol (given P. Schnewlin).

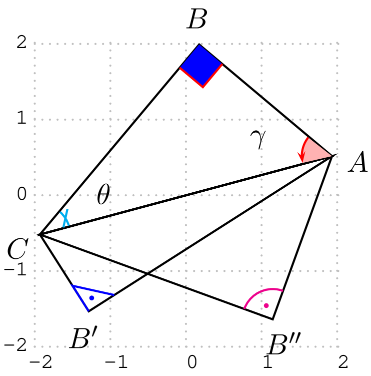
The optional argument `RightAngleSize` defines the size of the symbol (by default 0.28 unit).

For a right angle style `german` or `swissromand` the distance of the dot is preset to 0.5 (`german`) or 0.45 (`swissromand`), relative to the radius. However, it can be controlled by the optional argument `RightAngleDotDistance` which is preset to 1. A greater value moves the dot away from the reference point.

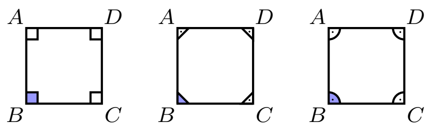
For other angles, there is the command:

`\pstMarkAngle [Options] {A}{B}{C}{Label}`

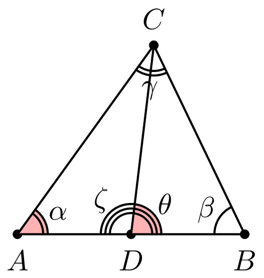
Valid optional arguments are `MarkAngleRadius`, `LabelAngleOffset`, `MarkAngleType` and `Mark`. The `label` can be any valid TEX box, it is put at `LabelSep` (by default 1 unit) of the node in the direction of the bisector of the angle modified by `LabelAngleOffset`(by default 0) and positioned using `LabelRefPt` (by default c). Furthermore the arc used for marking has a radius of `MarkAngleRadius` (by default .4 unit). At least, it is possible to place an arrow using the parameter `arrows`.Finally, it is possible to mark the angle by specifying a TEX command as argument of parameter `Mark`.




```
\begin{pspicture}[showgrid](-2,-2)(2,2)
\psset{PointSymbol=none}
\pstTriangle(2;15){A}(2;85){B}(2;195){C}
\psset{PointName=none}
\pstTriangle[PointNameA=default](2;-130){B'}(2;15){A'}(2;195){C'}
\pstTriangle[PointNameA=default](2;-55){B''}(2;15){A''}(2;195){C''}
\pstRightAngle[linecolor=red,fillstyle=solid,fillcolor=blue]{C}{B}{A}
\pstRightAngle[linecolor=blue, RightAngleType=suisseromand]{A}{B'}{C}
\pstRightAngle[linecolor=magenta, RightAngleType=german]{A}{B''}{C}
\psset{arcsep=\pslinewidth}
\pstMarkAngle[linecolor=cyan, Mark=MarkHash]{A}{C}{B}{$\theta$}
\pstMarkAngle[linecolor=red, arrows=->,fillcolor=red!30,
 fillstyle=solid]{B}{A}{C}{$\gamma$}
\end{pspicture}
```




```
\begin{pspicture}(-0.5,-0.5)(9,3)
\psset{PointSymbol=none,PointNameMathSize=\scriptstyle,PointNameSep=6pt,
   RightAngleSize=0.15,PosAngle={135,225,-45,45}}
\pstGeonode(1,2){A}(1,1){B}(2,1){C}(2,2){D}%
\pstRightAngle[fillstyle=solid,fillcolor=blue!40]{C}{B}{A}
\pstRightAngle{D}{C}{B} \pstRightAngle{A}{D}{C}
\pstRightAngle{B}{A}{D} \pspolygon(A)(B)(C)(D)
\psset{RightAngleType=suisseromand}
\pstGeonode(3,2){A}(3,1){B}(4,1){C}(4,2){D}%
\pstRightAngle[fillstyle=solid,fillcolor=blue!40]{C}{B}{A}
\pstRightAngle{D}{C}{B} \pstRightAngle{A}{D}{C}
\pstRightAngle{B}{A}{D} \pspolygon(A)(B)(C)(D)
\psset{RightAngleType=german}
\pstGeonode(5,2){A}(5,1){B}(6,1){C}(6,2){D}%
\pstRightAngle[fillstyle=solid,fillcolor=blue!40]{C}{B}{A}
\pstRightAngle{D}{C}{B} \pstRightAngle{A}{D}{C}
\pstRightAngle{B}{A}{D} \pspolygon(A)(B)(C)(D)
\end{pspicture}
```



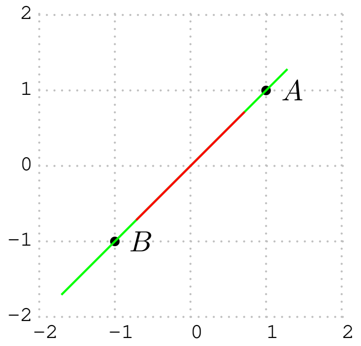

```
\begin{pspicture}[showgrid=false](-1.0,-1.0)(4,4)
\pstGeonode[PosAngle=-90](0.0,0.0){A}
\pstGeonode[PosAngle=-90](3.0,0.0){B}
\pstGeonode[PosAngle=90](1.8,2.5){C}
\pstGeonode[PosAngle=-90](1.5,0.0){D}
\pstMarkAngle[LabelSep=.6,MarkAngleRadius=.4,MarkAngleType=double]{A}{C}{B}{$\gamma$}
\pstMarkAngle[LabelSep=.6,MarkAngleRadius=.4,MarkAngleType=default]{C}{B}{A}{$\beta$}
\pstMarkAngle[LabelSep=.6,MarkAngleRadius=.4,MarkAngleType=double,fillcolor=red!30,fillstyle=solid]{B
    }{A}{C}{$\alpha$}
\pstMarkAngle[LabelSep=.6,MarkAngleRadius=.4,MarkAngleType=triple,fillcolor=red!30,fillstyle=solid]{B
    }{D}{C}{$\theta$}
\pstMarkAngle[LabelSep=.6,MarkAngleRadius=.4,MarkAngleType=triple]{C}{D}{A}{$\zeta$}
\pstLineAB{A}{B}
\pstLineAB{B}{C}
\pstLineAB{C}{A}
\pstLineAB{C}{D}
\end{pspicture}
```

2.6. Lines, half-lines and segments

The classical line $(\overline{AB})$!

```
\pstLineAB [Options] {A}{B}
```

In order to control its length[3], the two parameters `nodesepA` et `nodesepB` specify the abscissa of the extremity of the drawing part of the line. A negative abscissa specify an outside point, while a positive abscissa specify an internal point. If these parameters have to be equal, `nodesep` can be used instead. The default value of these parameters is equal to 0.




```
\begin{pspicture}[showgrid](-2,-2)(2,2)
\pstGeonode(1,1){A}(-1,-1){B}
\pstLineAB[nodesepA=-.4,nodesepB=-1,
        linecolor=green]{A}{B}
\pstLineAB[nodesep=.4,linecolor=red]{A}{B}
\end{pspicture}
```

The macro `\pstLine` draws a new line with two nodes, or two coordinates or one node and one coordinate. This macro is similar with `\pstLineAB`, but more compatible.

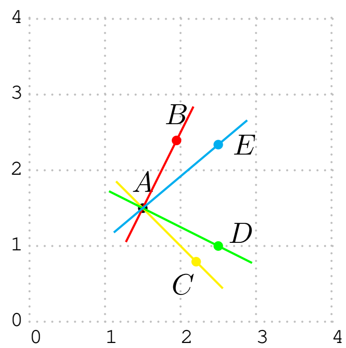
3 which is the comble for a line!

\pstLine [Options] $\{A\}\{B\}$
\pstLine [Options] $\{A\}(x,y)$
\pstLine [Options] $(x,y)\{B\}$
\pstLine [Options] $(x,y)(x,y)$

The macros \pstLineAA and \pstLineAS draw a new line with one node, the slope angle between the line and the horizontal axis, or the slope gradient of the line, and create a new node B on the line.

\pstLineAA [Options] $\{A\}\{angle\}\{B\}$
\pstLineAA [Options] $(x,y)\{angle\}\{B\}$
\pstLineAS [Options] $\{A\}\{gradient\}\{B\}$
\pstLineAS [Options] $(x,y)\{gradient\}\{B\}$

Here are some examples:

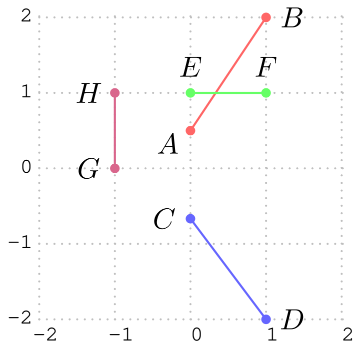



```
\begin{pspicture}[showgrid=true](0,0)(4,4)
\pstGeonode[PosAngle=90](1.5,1.5){A}
% draw a line with angle atan(2/1), about 63.43 degree.
\pstLineAA[linecolor=red,nodesep=-0.5,PosAngle=90]{A}{2 1 atan}{B
    }
\pstLineAA[linecolor=yellow,nodesep=-0.5,PosAngle=-120]{A}{-45}{C
    }
\pstLineAS[linecolor=green,nodesep=-0.5,PosAngle=30]{A}{-0.5}{D}
% draw a line with gradient (cos50/sin50).
\pstLineAS[linecolor=cyan,nodesep=-0.5]{A}{50 cos 50 sin div}{E}
\end{pspicture}
```

The macros \pstLineCoef is used to draw a line $ax + by + c = 0$ with the given coefficents a, b, c, and create two new node A, B on the line.

\pstLineCoef [Options] $\{a,b,c\}\{A\}\{B\}$

Here are some examples:

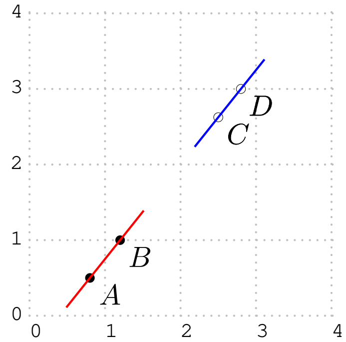



```
\begin{pspicture}[showgrid=true](-2,-2)(2,2)
\pstLineCoef[linecolor=red!60, PosAngle={210,0}]{3,-2,1}{A}{B}
\pstLineCoef[linecolor=blue!60, PosAngle={180,0}]{4,3,2}{C}{D}
\pstLineCoef[linecolor=green!60, PosAngle={90,90}]{0,3,-3}{E}{F}
\pstLineCoef[linecolor=purple!60, PosAngle={180,180}]{4,0,4}{G}{H
    }
\end{pspicture}
```

The macro \pstLineAbsNode creates a new node C whose abscissa is the given value x_1 on the line AB. The macro \pstLineOrdNode creates a new node C whose ordinate is the given value y_1 on the line AB. You can input x_1 or y_1 as any number(e.g, 2.0), or use \pscalculate or \fpeval to get a purely numerical result, or use \pstAbscissa and \pstOrdinate to get the abscissa and ordinate of any other node.

\pstLineAbsNode [Options] $\{A\}\{B\}\{x_1\}\{C\}$
\pstLineOrdNode [Options] $\{A\}\{B\}\{y_1\}\{C\}$

For example,



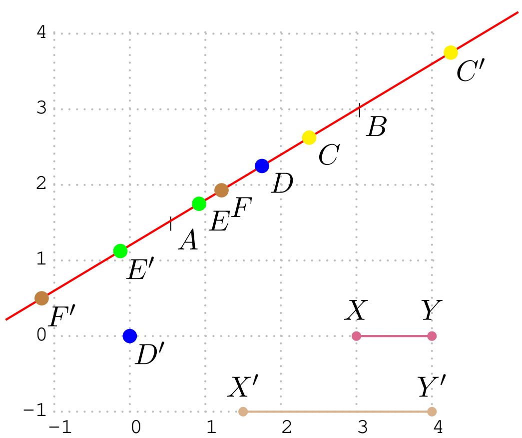

```
\begin{pspicture}[showgrid=true](0,0)(4,4)
\pstGeonode[PosAngle=-40](0.8,0.5){A}
\pstGeonode[PosAngle=-40](1.2,1.0){B}
\pstLineAB[linecolor=red,nodesep=-0.5]{A}{B}
\pstLineAbsNode[PosAngle=-40,PointSymbol=o]{A}{B}{2.5}{C}
\pstLineOrdNode[PosAngle=-40,PointSymbol=o]{A}{B}{3.0}{D}
\pstLineAB[linecolor=blue,nodesep=-0.5]{C}{D}
\end{pspicture}
```

The macro \pstProportionNode creates the nodes C and C' on segment AB which are satisfied $|AC| : |BC| = \lambda$, $(\lambda > 0)$. The node C is inside the segment AB and the node C' is outside the segment AB, we have

$$\begin{cases} x_C = \dfrac{x_A + \lambda x_B}{1+\lambda} \\ y_C = \dfrac{y_A + \lambda y_B}{1+\lambda} \end{cases} \quad \text{and} \quad \begin{cases} x_{C'} = \dfrac{x_A - \lambda x_B}{1-\lambda} \\ y_{C'} = \dfrac{y_A - \lambda y_B}{1-\lambda} \end{cases}$$

\pstProportionNode [Options] $\{A\}\{B\}\{\lambda\}\{C\}\{C'\}$

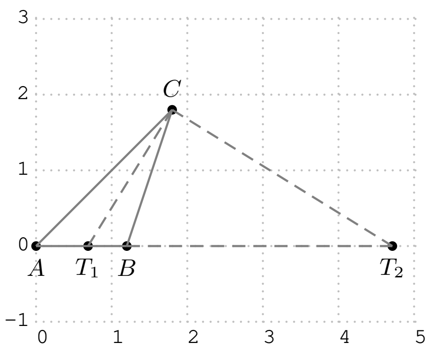
You can use \pstDistDiv to get the ratio of two segments to λ, we will introduce \pstDistDiv later.




```
\begin{pspicture}[showgrid=true](-1,-1)(4,4)
\pstGeonode[PosAngle=-40,PointSymbol=|](0.5,1.5){A}
\pstGeonode[PosAngle=-40,PointSymbol=|](3.0,3.0){B}
\pstGeonode[PosAngle=90,linecolor=purple!60,CurveType=polyline
    ](3,0){X}(4,0){Y}
\pstGeonode[PosAngle=90,linecolor=brown!60,CurveType=polyline
    ](1.5,-1){X'}(4,-1){Y'}
\pstLineAB[linecolor=red,nodesep=-2.5]{A}{B}
\psset{PosAngle=-40,PointSymbol=*,dotscale=1.5}
\pstProportionNode[linecolor=yellow]{A}{B}{3.0}{C}{C'}
\pstProportionNode[linecolor=blue]{A}{B}{1.0}{D}{D'}
\pstProportionNode[linecolor=green]{A}{B}{0.2}{E}{E'}
\pstProportionNode[linecolor=brown]{A}{B}{\pstDistDiv{X}{Y}{X'}{Y
    '}}{F}{F'}
\end{pspicture}
```

One application of \pstProportionNode is used to find the bisector and out bisector of a given angle. So we define the macro \pstBisectorAOB to do this work, it is more friendly than the macros \pstBissectBAC and \pstOutBissectBAC, as it put the new node T_1 and T_2 on line AB, not arc AB.

\pstBisectorAOB [Options] $\{A\}\{O\}\{B\}\{T_1\}\{T_2\}$



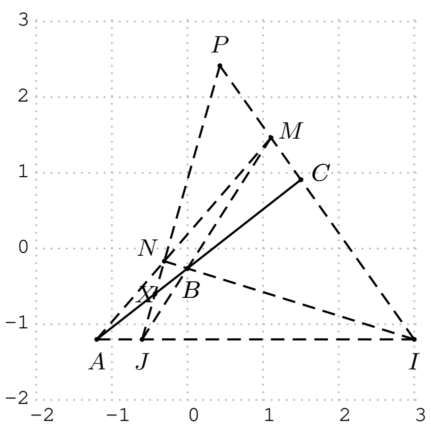

```
\begin{pspicture}[showgrid=true](0,-1)(5,3)
\psset{unit=0.6cm}\footnotesize\psset{PointSymbol=*}
\pstGeonode[PosAngle=-90](0,0){A}
\pstGeonode[PosAngle=90](3,3){C}
\pstGeonode[PosAngle=-90](2,0){B}
\pstBisectorAOB[PosAngle={-90,-90}]{A}{C}{B}{T_1}{T_2}
\pstLineAB[linecolor=black!50]{A}{C}
\pstLineAB[linecolor=black!50]{B}{C}
\pstLineAB[linecolor=black!50]{A}{B}
\pstLineAB[linestyle=dashed,linecolor=gray]{C}{T_1}
\pstLineAB[linestyle=dashed,linecolor=gray]{C}{T_2}
\pstLineAB[linestyle=dashed,linecolor=gray]{B}{T_2}
\end{pspicture}
```

The four collinear points A, B, C, D are called `Harmonic Conjugation Points` if their cross ratio is -1, that is

$$(AB, CD) = \frac{AC}{BC} : \frac{AD}{BD} = -1$$

If given three collinear points A, B, C, how can we get the fourth harmonic point? The following macro `\pstFourthHarmonicNode` is used to get the fourth harmonic point. It create a new node X on the same line, but when A, B, C are not collinear, we put it at origin.

`\pstFourthHarmonicNode [Options]` $\{A\}\{B\}\{C\}\{X\}$

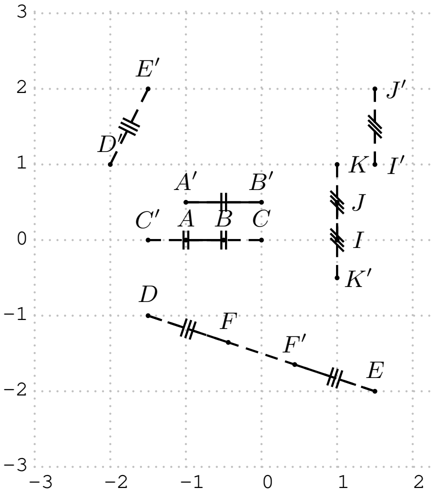



```
\begin{pspicture}[showgrid=true](-2,-2)(3,3)\footnotesize
\psset{unit=0.6cm}\psset{dotscale=0.5}\psset{PointSymbol=*}
\pstGeonode[PosAngle=-90](-2,-2){A}(5,-2){I}(-1,-2){J}
\pstLineAA[linestyle=none,PointName=none,PointSymbol=none]{A
    }{38}{A'}
\pstLineAbsNode[PosAngle=-80]{A}{A'}{0}{B}
\pstLineAbsNode[PosAngle=20]{A}{A'}{2.5}{C}
\pstFourthHarmonicNode[PosAngle=180,PointNameSep=0.2]{A}{B}{C}{X}
% check if A,N,M are also collinear.
\pstInterLL[PosAngle=90]{J}{X}{I}{C}{P}
\pstLineAB[linestyle=dashed]{J}{P}
\pstLineAB[linestyle=dashed]{I}{P}
\pstInterLL[PosAngle=20]{J}{B}{I}{P}{M}
\pstInterLL[PosAngle=140]{I}{B}{J}{P}{N}
\pstLineAB[linestyle=dashed]{J}{M}
\pstLineAB[linestyle=dashed]{I}{N}
\pstLineAB[linestyle=dashed]{A}{M}
\pstLineAB[linestyle=dashed]{A}{I}
\pstLineAB{A}{C}
\end{pspicture}
```

If you want to draw a node like 'Given EF, please find node C on AB such that $AC = EF$', you can use the macro `\pstLocateAB` to do this, it can seek the node C from A to B with the specified length L, which can be got from `\pstDist`, `\pstDistConst`, `\pstDistAdd`, `\pstDistSub`, etc.

`\pstLocateAB [Options]` $\{A\}\{B\}\{L\}\{C\}$

Note that seek from B will get the node C in the reverse order, for example,

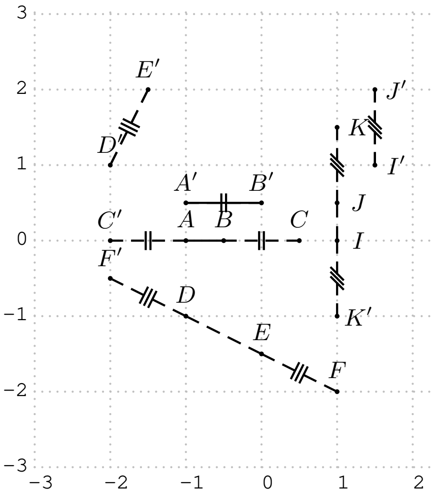



```
\begin{pspicture}[showgrid=true](-3,-3)(3,3)\footnotesize
\psset{unit=0.5cm}\psset{dotscale=0.5}\psset{PointSymbol=*}
\pstGeonode[PosAngle=90,CurveType=polyline](-2,0){A}(-1,0){B}
\pstGeonode[PosAngle=90,CurveType=polyline](-2,1){A'}(0,1){B'}
\pstLocateAB[PosAngle=90]{A}{B}{\pstDist{A'}{B'}}{C}
\pstLocateAB[PosAngle=90]{B}{A}{\pstDist{A'}{B'}}{C'}
\psset{linestyle=dashed,SegmentSymbol=MarkHashh,MarkAngle=90}
\pstSegmentMark{A'}{B'}\pstSegmentMark{B}{C'}\pstSegmentMark{A}{C}
\pstGeonode[PosAngle=90,CurveType=polyline](-3,-2){D}(3,-4){E}
\pstGeonode[PosAngle=90,CurveType=polyline](-4,2){D'}(-3,4){E'}
\pstLocateAB[PosAngle=90]{D}{E}{\pstDist{D'}{E'}}{F}
\pstLocateAB[PosAngle=90]{E}{D}{\pstDist{D'}{E'}}{F'}
\psset{linestyle=dashed,SegmentSymbol=MarkHashhh,MarkAngle=90}
\pstSegmentMark{D'}{E'}\pstSegmentMark{E}{F'}\pstSegmentMark{D}{F}
\pstGeonode[PosAngle=0,CurveType=polyline](2,0){I}(2,1){J}
\pstGeonode[PosAngle=0,CurveType=polyline](3,2){I'}(3,4){J'}
\pstLocateAB[PosAngle=0]{I}{J}{\pstDist{I'}{J'}}{K}
\pstLocateAB[PosAngle=0]{J}{I}{\pstDist{I'}{J'}}{K'}
\psset{linestyle=dashed,SegmentSymbol=MarkHashhh,MarkAngle=45}
\pstSegmentMark{I'}{J'}\pstSegmentMark{J}{K'}\pstSegmentMark{I}{K}
\end{pspicture}
```

If you want to draw a node like 'Given EF, please extend AB to C such that $BC = EF$', you can use the macro `\pstExtendAB` to do this, it can extend AB from B to one node with the specified length L, which can be got from `\pstDist`, `\pstDistConst`, `\pstDistAdd`, `\pstDistSub`, etc.

`\pstExtendAB [Options]` $\{A\}\{B\}\{L\}\{C\}$

Note that extend BA to C will get the node C in the reverse order, for example,




```
\begin{pspicture}[showgrid=true](-3,-3)(3,3)\footnotesize
\psset{unit=0.5cm}\psset{dotscale=0.5}\psset{PointSymbol=*}
\pstGeonode[PosAngle=90,CurveType=polyline](-2,0){A}(-1,0){B}
\pstGeonode[PosAngle=90,CurveType=polyline](-2,1){A'}(0,1){B'}
\pstExtendAB[PosAngle=90]{A}{B}{\pstDist{A'}{B'}}{C}
\pstExtendAB[PosAngle=90]{B}{A}{\pstDist{A'}{B'}}{C'}
\psset{linestyle=dashed,SegmentSymbol=MarkHashh,MarkAngle=90}
\pstSegmentMark{A'}{B'}\pstSegmentMark{B}{C}\pstSegmentMark{A}{C'}
\pstGeonode[PosAngle=90,CurveType=polyline](-2,-2){D}(0,-3){E}
\pstGeonode[PosAngle=90,CurveType=polyline](-4,2){D'}(-3,4){E'}
\pstExtendAB[PosAngle=90]{D}{E}{\pstDist{D'}{E'}}{F}
\pstExtendAB[PosAngle=90]{E}{D}{\pstDist{D'}{E'}}{F'}
\psset{linestyle=dashed,SegmentSymbol=MarkHashhh,MarkAngle=90}
\pstSegmentMark{D'}{E'}\pstSegmentMark{E}{F}\pstSegmentMark{D}{F'}
\pstGeonode[PosAngle=0,CurveType=polyline](2,0){I}(2,1){J}
\pstGeonode[PosAngle=0,CurveType=polyline](3,2){I'}(3,4){J'}
\pstExtendAB[PosAngle=0]{I}{J}{\pstDist{I'}{J'}}{K}
\pstExtendAB[PosAngle=0]{J}{I}{\pstDist{I'}{J'}}{K'}
\psset{linestyle=dashed,SegmentSymbol=MarkHashhh,MarkAngle=45}
\pstSegmentMark{I'}{J'}\pstSegmentMark{J}{K}\pstSegmentMark{I}{K'}
\end{pspicture}
```

You can find the node C on segment AB satisfied $|AC|$:$|AB|$=`DistCoef` using `\pstTranslation`, but it can't do the same thing like `\pstLocateAB` and `\pstExtendAB` when the given segment EF is not parallel with AB, it will be introduced in the later sections.

If you want to find the inversion point C' of C to the inversion center O with inversion raduis R, that is, the point C' is satisfied the inversion transform equation

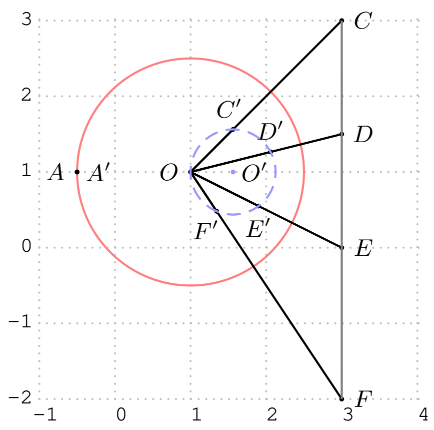
$$|OC| \times |OC'| = R^2$$

you can use the macro `\pstInversion` to do this work. In fact, we use the macro `\pstLocateAB` to implement this macro by passing the value $\dfrac{R^2}{|OC|}$ to parameter length.

`\pstInversion [Options]` $\{O\}\{A\}\{C\}\{C'\}$

It is possible to omit the parameter A and then to specify the inversion radius or the inversion diameter using the parameters `Radius` and `Diameter`, which will be introduced in the next section.

It is clear that the inversion mapping of a line is a circle, and the inversion mapping of a point on the inversion circle is itself.




```
\begin{pspicture}[showgrid=true](-1,-2)(4,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\ra{1.5}
\pstGeonode[PosAngle=180](1,1){O}
\pstCircleOA[linecolor=red!50,Radius=\pstDistVal{\ra}]{O}{}
\pstCircleRotNode[PosAngle=180,RotAngle=180,Radius=\pstDistVal{\
    ra}]{O}{}{A}
\pstInversion[PosAngle=0,Radius=\pstDistVal{\ra}]{O}{}{A}{A'}
\pstGeonode[PosAngle=0](3,3){C}
\pstInversion[PosAngle=100,Radius=\pstDistVal{\ra}]{O}{}{C}{C'}
\pstLineAB{O}{C}\pstLineAB{O}{C'}
\pstGeonode[PosAngle=0](3,1.5){D}
\pstInversion[PosAngle=90,Radius=\pstDistVal{\ra}]{O}{}{D}{D'}
\pstLineAB{O}{D}\pstLineAB{O}{D'}
\pstGeonode[PosAngle=0](3,0){E}
\pstInversion[PosAngle=-90,Radius=\pstDistVal{\ra}]{O}{}{E}{E'}
\pstLineAB{O}{E}\pstLineAB{O}{E'}
\pstGeonode[PosAngle=0](3,-2){F}
\pstInversion[PosAngle=-120]{O}{A}{F}{F'}
\pstLineAB{O}{F}\pstLineAB{O}{F'}
\pstLineAB[linecolor=black!50]{C}{F}
\pstCircleABC[linestyle=dashed,linecolor=blue!40,PosAngle=0]{C'}{
    D'}{E'}{O'}
\end{pspicture}
```

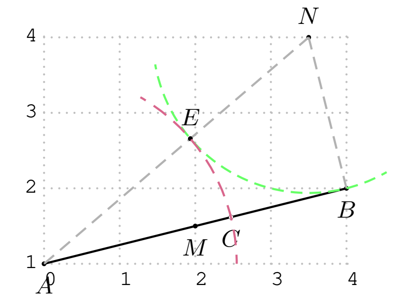
If you want to find the node C from A to B, such that C is the golden section of the given segments AB, that is,

$$|AC|^2 = |AB| \times |BC| \quad \text{or} \quad AC : AB = BC : AC \quad \text{or} \quad AC = \frac{\sqrt{5}-1}{2}AB$$

you can use the macro `\pstGoldenMean` to do this work.

`\pstGoldenMean [Options]` $\{A\}\{B\}\{C\}$

In fact, we use the macro `\pstLocateAB` to implement this macro by passing the value $\dfrac{\sqrt{5}-1}{2}|AB|$ to parameter length.




```
\begin{pspicture}[showgrid=true](0,1)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\pstGeonode[PosAngle=-90,CurveType=polyline](0,1){A}(4,2){B}
\pstGoldenMean[PosAngle=-90,PointSymbol=o]{A}{B}{C}
% geometrical method to draw the golden point
\pstMiddleAB[PosAngle=-90]{A}{B}{M}
\pstRotation[RotAngle=-90,PosAngle=90]{B}{M}[N]
\pstLineAB[linestyle=dashed,linecolor=gray!60]{A}{N}
\pstLineAB[linestyle=dashed,linecolor=gray!60]{B}{N}
\pstInterLC[PointNameA=,PosAngleB=90]{N}{A}{N}{B}{B0}{E}
\pstCircleOA[linecolor=green!60,linestyle=dashed]{N}{B}[190][300]
\pstCircleOA[linecolor=purple!60,linestyle=dashed]{A}{E}[0][60]
\end{pspicture}
```

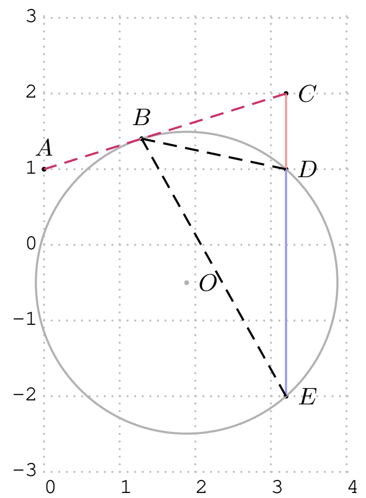
If you want to find the node C from A to B, such that AC is the geometric mean of two given segments DE of FG, that is,

$$|AC|^2 = |DE| \times |FG|$$

you can use the macro `\pstGeometricMean` to do this work. It also can be used to draw a circle when given two points on the circle, and a line tangents to the circle.

`\pstGeometricMean [Options]` $\{A\}\{B\}\{L_1\}\{L_2\}\{C\}$

In fact, we use the macro `\pstLocateAB` to implement this macro by passing the value $\sqrt{L_1 \times L_2}$ to parameter length. The length L_1 and L_2 can be got from `\pstDist`, `\pstDistConst`, `\pstDistAdd`, `\pstDistSub`, etc.




```
\begin{pspicture}[showgrid=true](0,-3)(4,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\pstGeonode[PosAngle=90](0,1){A}
\pstGeonode[PosAngle=0](3.2,2){C}(3.2,1){D}(3.2,-2){E}
\pstGeometricMean[PosAngle=90]{C}{A}{\pstDistAB{C}{D}}{\pstDistAB
    {C}{E}}{B}
\pstCircleABC[linecolor=gray!60]{B}{D}{E}{O}
\pstLineAB[linecolor=red!40]{C}{D}
\pstLineAB[linecolor=blue!40]{D}{E}
\psset{linestyle=dashed}
\pstLineAB[linecolor=purple!80]{C}{A}
\pstLineAB{D}{B}\pstLineAB{E}{B}
\end{pspicture}
```

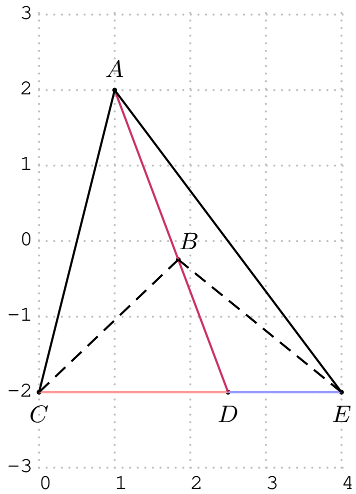
If you want to find the node C from A to B, such that AC is the harmonic mean of two given segments DE of FG, that is,

$$\frac{1}{|AC|} = \frac{1}{2}\left(\frac{1}{|DE|} + \frac{1}{|FG|}\right)$$

you can use the macro `\pstHarmonicMean` to do this work.

`\pstHarmonicMean [Options]` $\{A\}\{B\}\{L_1\}\{L_2\}\{C\}$

In fact, we use the macro `\pstLocateAB` to implement this macro by passing the value $\frac{2L_1L_2}{L_1+L_2}$ to parameter length. The length L_1 and L_2 can be got from `\pstDist`, `\pstDistConst`, `\pstDistAdd`, `\pstDistSub`, etc.




```
\begin{pspicture}[showgrid=true](0,-3)(4,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\pstGeonode[PosAngle=90](1,2){A}
\pstGeonode[PosAngle=-90](0,-2){C}(2.5,-2){D}(4,-2){E}
\pstHarmonicMean[PosAngle=60]{D}{A}{\pstDistAB{C}{D}}{\pstDistAB{
    D}{E}}{B}
\pstLineAB[linecolor=red!40]{C}{D}
\pstLineAB[linecolor=blue!40]{D}{E}
\pstLineAB[linecolor=purple!80]{A}{D}
\pstLineAB[linestyle=dashed]{C}{B}
\pstLineAB[linestyle=dashed]{E}{B}
\pstLineAB{C}{A}\pstLineAB{E}{A}
\end{pspicture}
```

2.7. Distance

Like as coordinates, the distance works at the PostScript level, that is, it should be used where the code is interpreted by PostScript engine, but not TEX engine. There were three macros to operate the distance before v1.66:

`\pstDistAB{`*A*`}{`*B*`}`
`\pstDistVal{`*l*`}`
`\pstDistCalc{`*expr*`}`

The first specifies a distance between two points. The second macro can be used to specify an explicit numerical value l, which is in `User coordinate`. The third one uses the `\pscalculate` to calculate the result of the input expression, which is in `User coordinate` too. The parameter `DistCoef` can be used to specify a coefficient to reduce or enlarge the result distance. This parameter will come into effect if it is specified before these macros.

After v1.66, We provide three macros which disable the effect of parameter `DistCoef` one to one as following:

`\pstDist{`*A*`}{`*B*`}`
`\pstDistConst{`*l*`}`
`\pstDistExpr{`*expr*`}`

We provide the macro `\pstDistCoef` to reduce or enlarge a given distance explicitly, for example: `\pstDistCoef{\pstDist{A}{B}}`, or use macro `\pstDistMul` to multiply the input coefficient.

Note: The series of macros `\pstDist*` get the length result in the `Screen coordinate`, so you need to convert the length to the `User coordinate` by macro `\pstUserDist`, when use them where need the user coordinate numbers, e.g,

```
\pnode(! 1 \pstUserDist{\pstDistAdd{A}{B}{C}{D}}){A}
\pstMoveNode(0,\pstUserDist{\pstDistAdd{A}{B}{C}{D}}){A}{E}
```

You can convert the distance in `User coordinate` to `Screen coordinate` by macro `\pstScreenDist`, it is just another name of `\pstDistConst`. As we said before, macros `\pstAbscissa` and `\pstOrdinate` give the coordinate of one node in `User coordinate`, so if you want to draw a circle using them, you should type:

```
\pstCircleOA[Radius=\pstDistConst{\pstAbscissa{A}}]{A}{}
```

It is possible to use the raw PostScript command to make more complex arithmetic operations. In order to hide the lower level Postscipt language, we add more macros for distance addition and subtraction, such as `\pstDistAdd`[Val/Coef] and `\pstDistSub`[Val/Coef], etc. These macros can be used to calculate the Radius or Diameter to define a circle.

The macros `\pstDistAdd` and `\pstDistSub` are used to get the addition and subtraction of the given segments AB and CD. The macro `\pstDistDiv` is used to get the length ratio of the given segments AB and CD, you can pass the ratio to macro `\pstProportionNode`, or setup the ratio to parameter `DistCoef` in macro `\pstTranslation`, or pass the ratio to any `\pstDist*` macros which need a λ parameter.

`\pstDistMul`{A}{B}{λ}
`\pstDistAdd`{A}{B}{C}{D}
`\pstDistAddVal`{A}{B}{λ}{L}
`\pstDistAddCoef`{A}{B}{λ_1}{C}{D}{λ_2}
`\pstDistSub`{A}{B}{C}{D}
`\pstDistSubVal`{A}{B}{λ}{L}
`\pstDistSubCoef`{A}{B}{λ_1}{C}{D}{λ_2}
`\pstDistDiv`{A}{B}{C}{D}

In these macros, the length L is a numerical value in the `Screen Coordinate`, so it is possible to pass the result of any macros like `\pstDist` to it. λ is a numerical value to multiply, and most important is that the parameter `DistCoef` doesn't take effect any more. It is better to describe in formula:
- macro `\pstDistAB` get the screen length of DistCoef $* |AB|$
- macro `\pstDistVal` get the screen length of DistCoef $* l$
- macro `\pstDistCalc` get the screen length of DistCoef $*$ expr
- macro `\pstDistCoef` get the screen length of DistCoef $*$ <arg>
- macro `\pstDist` get the screen length of $|AB|$
- macro `\pstDistConst` get the screen length of l
- macro `\pstDistExpr` get the screen length of expr
- macro `\pstDistMul` get the screen length of $\lambda|AB|$
- macro `\pstDistAdd` get the screen length of $|AB| + |CD|$
- macro `\pstDistAddVal` get the screen length of $\lambda|AB| + L$
- macro `\pstDistAddCoef` get the screen length of $\lambda_1|AB| + \lambda_2|CD|$
- macro `\pstDistSub` get the screen length of $abs(|AB| - |CD|)$
- macro `\pstDistSubVal` get the screen length of $abs(\lambda|AB| - L)$
- macro `\pstDistSubCoef` get the screen length of $abs(\lambda_1|AB| - \lambda_2|CD|)$
- macro `\pstDistDiv` get the the ratio of length $|AB| : |CD|$

For example, the following one draw a circle with radius length $2|AB| + 3|CD| + 4|EF|$, it shows how to operate more than two distances.

```
\pstCircleOA[Radius=\pstDistAddVal{A}{B}{2.0}{\pstDistAddCoef{C}{D}{3.0}{E}{F}{4.0}}]{A}{}
```

Another example is for `\pstDistMul`, the old code like as

```
\pstCircleOA[DistCoef=1 3 div,Radius=\pstDistAB{A}{B}]{O}{}
\pstCircleOA[DistCoef=1 3 div,Radius=\pstDistAB{A}{B}]{A}{B}{O}{}{I}{J}
\pstInterCC[DistCoef=1 3 div,RadiusA=\pstDistAB{A}{B},DistCoef=none,RadiusB=\pstDistAB{C}{D}]{O1}{}{
    O2}{}{I}{J}
```

could be simplified to

```
\pstCircleOA[Radius=\pstDistMul{A}{B}{1 3 div}]{O}{}
```

```
\pstInterLC[Radius=\pstDistMul{A}{B}{1 3 div}]{A}{B}{O}{}{I}{J}
\pstInterCC[RadiusA=\pstDistMul{A}{B}{1 3 div},RadiusB=\pstDistAB{C}{D}]{O1}{}{O2}{}{I}{J}
```

Important! We recommend that you should use the distance macros which disable the parameter DistCoef instead of \pstDistAB, \pstDistVal or \pstDistCalc, when you need to pass their result into \pstDistAddVal or \pstDistSubVal, as it will give you the error result sometimes. For example, the following code

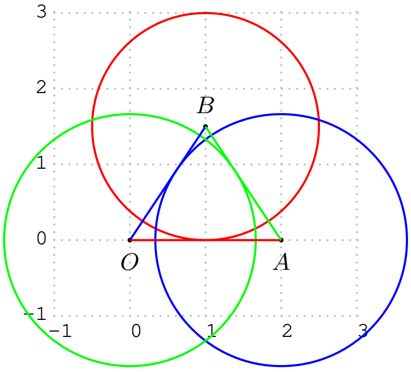
```
\pstDistAddVal{A}{B}{2.0}{\pstDistAB{C}{D}}
```

is expected to get the length of $2|AB| + |CD|$. If current DistCoef is λ, then it will give the error result as $2|AB| + \lambda|CD|$. The right way is

```
\pstDistAddVal{A}{B}{2.0}{\pstDist{C}{D}}
```

At last, we provide a macro named \pstDistABC to get the distance from C to line AB.

\pstDistABC{A}{B}{C}




```
\begin{pspicture}[showgrid=true](-1,-1)(3,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\pstGeonode[PosAngle=-90](0,0){O}
\pstGeonode[PosAngle=-90](2,0){A}
\pstGeonode[PosAngle=90](1,1.5){B}
\pstCircleOA[linecolor=red,Radius=\pstDistABC{O}{A}{B}]{B}{}
\pstCircleOA[linecolor=blue,Radius=\pstDistABC{B}{O}{A}]{A}{}
\pstCircleOA[linecolor=green,Radius=\pstDistABC{A}{B}{O}]{O}{}
\pstLineAB[linecolor=red]{O}{A}
\pstLineAB[linecolor=blue]{B}{O}
\pstLineAB[linecolor=green]{A}{B}
\end{pspicture}
```

2.8. Circles

A circle can be defined either with its center and a point of its circumference, or with two diameterly opposed points. There are two commands:

\pstCircleOA [Options] {O}{A} [angleA] [angleB]
\pstCircleAB [Options] {A}{B} [angleA] [angleB]

\pstCircleOA draws the circle of center O crossing A from angleA to angleB, going counter clockwise. Possible options are Radius and Diameter.

\pstCircleAB draws the circle of diameter AB with the same options.

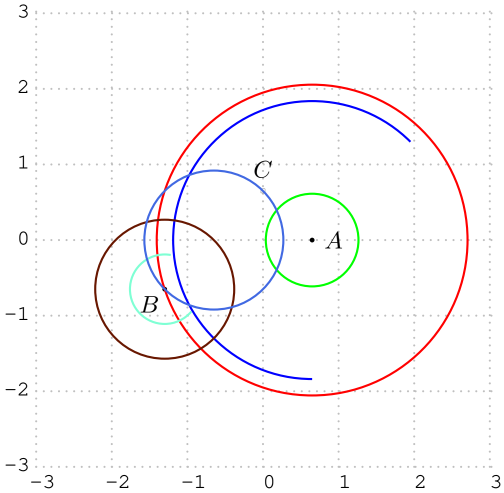
For the first macro, it is possible to omit the second point and then to specify a radius or a diameter using the parameters Radius [4] and Diameter. The values of these parameters can be specified with one of the \pstDist* series macros.

We will see later how to draw the circle crossing three points. With this package, it becomes possible to draw:

- the circle of center A crossing B;
- the circle of center A whose radius is AC;
- the circle of center A whose radius is BC;

4 The package pst-fractal also defines an optional key named Radius, if you need to use this package with pst-eucl, you need to setup the key Radius as following: \psset[pst-eucl]{Radius=\pstDistVal{3}}.

- the circle of center B whose radius is AC;
- the circle of center B of diameter AC;
- the circle whose diameter is BC.

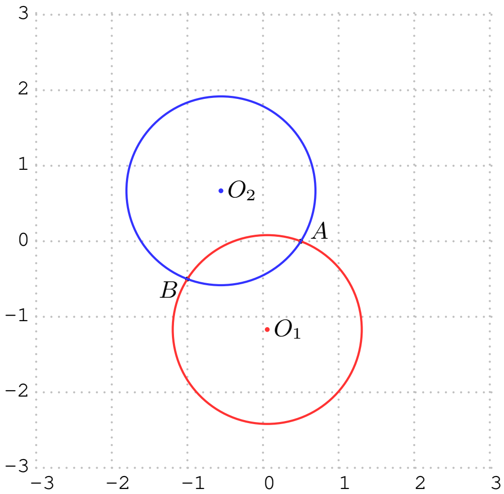



```
\begin{pspicture}[showgrid](-3,-3)(3,3)\footnotesize
\psset{unit=0.65cm}\psset{dotscale=0.5}\psset{PointSymbol=*}
\pstGeonode[PosAngle={0,-135,90},PointSymbol={*,*,square}](1,0){A
    }(-2,-1){B}(0,1){C}
\pstCircleOA[linecolor=red]{A}{B}
\pstCircleOA[linecolor=green, Radius=\pstDistMul{A}{C}{2 3 div}]{
    A}{}
\pstCircleOA[linecolor=blue, Radius=\pstDistAB{B}{C}]{A
    }{}[45][270]
\pstCircleOA[linecolor=Sepia, Radius=\pstDistAB{A}{C}]{B}{}
\pstCircleOA[linecolor=Aquamarine, Diameter=\pstDistAB{A}{C}]{B
    }{}[80][320]
\pstCircleAB[linecolor=RoyalBlue]{B}{C}
\end{pspicture}
```

The macro `\pstCircleABR` draws the circle of given radius length R, through two given nodes A and B, then outputs the circle center O.

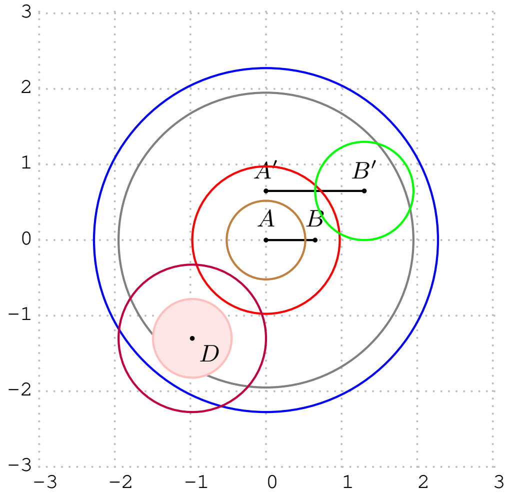
`\pstCircleABR [Options]` $\{A\}\{B\}\{R\}\{O\}$

Note that through from A to B and through from B to A will get the figure symmetric to AB. For example,




```
\begin{pspicture}[showgrid](-3,-3)(3,3)\footnotesize
\psset{unit=0.50cm}\psset{dotscale=0.5}\psset{PointSymbol=*}
\pstGeonode[PosAngle={30,210}](1,0){A}(-2,-1){B}
\pstCircleABR[linecolor=red!80]{A}{B}{\pstDistConst{2.5}}{O_1}
\pstCircleABR[linecolor=blue!80]{B}{A}{\pstDistConst{2.5}}{O_2}
\end{pspicture}
```

The following example show how to use the more complex distance macros, and the parameter to fill the circle.



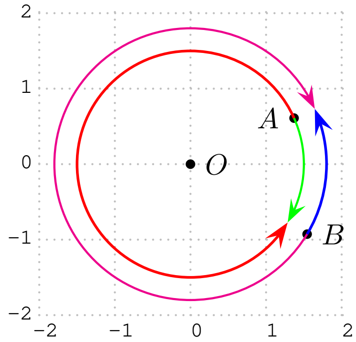

```
\begin{pspicture}[showgrid=true](-3,-3)(3,3)\footnotesize
\psset{unit=0.65cm}\psset{dotscale=0.5}\psset{PointSymbol=*}
\pstGeonode[PosAngle=90,CurveType=polyline](0,0){A}(1,0){B}
\pstGeonode[PosAngle=90,CurveType=polyline](0,1){A'}(2,1){B'}
\pstCircleOA[linecolor=gray,Radius=\pstDistAdd{A}{B}{A'}{B'}]{A
    }{} % R=|AB|+|A'B'|
\pstCircleOA[linecolor=red,Radius=\pstDistAddVal{A}{B}{1.0}{\
    pstDistConst{0.5}}]{A}{} % R=|AB|+0.5
\pstCircleOA[linecolor=blue,Radius=\pstDistAddCoef{A}{B}{0.5}{A
    '}{B'}{1.5}]{A}{} % R=0.5|AB|+1.5|A'B'|
\pstCircleOA[linecolor=green,Radius=\pstDistSub{A}{B}{A'}{B'}]{B
    '}{} % R=|AB|-|A'B'|
\pstCircleOA[linecolor=brown,Radius=\pstDistSubCoef{A}{B}{1.8}{A
    '}{B'}{0.5}]{A}{} % R=1.8|AB|-0.5|A'B'|
\pnode(-1.5,-2){D}
\pstCircleOA[linecolor=pink,fillstyle=solid,fillcolor=pink!40,
    Radius=\pstDistMul{A}{B}{0.8}]{D}{} % R=0.8|AB|
\psdot(D)\uput{0.2}[-45](D){$D$}
\pstCircleOA[linecolor=purple,Radius=\pstDistConst{\pstAbscissa{D
    }} abs]{D}{} % R=|D.x|
\end{pspicture}
```

The last row set the absolute value of the abscissa of node D to `Radius`, and then draw a circle at center D. Note that it does not work before v1.67, as the `\pstCircleOA` and `\pstCircleAB` were implemented with a `\rput` command, which will set the center D's coordinate to origin, it causes that the Radius was set to zero and none circle will be draw out, so we remove the `\rput` code in v1.67, and everything works well now.

2.9. Circle arcs

`\pstArcOAB [Options] {`O`}{`A`}{`B`}`
`\pstArcnOAB [Options] {`O`}{`A`}{`B`}`

These two macros draw circle arcs, O is the center, the radius defined by OA, the beginning angle given by A and the final angle by B. Finally, the first macro draws the arc in the direct way, whereas the second in the indirect way. It is not necessary that the two points are at the same distance of O.



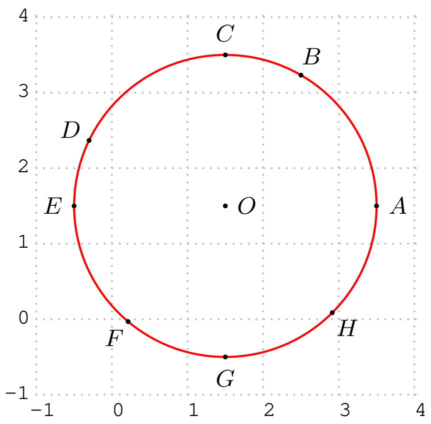

```
\begin{pspicture}[showgrid](-2,-2)(2,2)
\pstGeonode[PosAngle={180,0}](1.5;24){A}(1.8;-31){B}
\pstGeonode{O}
\psset{arrows=->,arrowscale=2}
\pstArcOAB[linecolor=red,linewidth=1pt]{O}{A}{B}
\pstArcOAB[linecolor=blue,linewidth=1pt]{O}{B}{A}
\pstArcnOAB[linecolor=green]{O}{A}{B}
\pstArcnOAB[linecolor=magenta]{O}{B}{A}
\end{pspicture}
```

2.10. Circle nodes

Do you want to draw a point on the circle? A point can be positioned on a circle using its rotation angle by macro `\pstCircleNode` or `\pstCircleRotNode`. The first `\pstCircleNode` requires an explicit parameter angle θ to calculate the point; but the second `\pstCircleRotNode` requires an implicit parameter `RotAngle` to calculate the point, If you not set `RotAngle`, the default value is 60°.

The circle is defined by center O and point A on the circle or `Radius` or `Diameter` in parameter.

```
\pstCircleNode [Options] {O}{A}{θ}{X}
\pstCircleRotNode [Options] {O}{A}{X}
```




```
\begin{pspicture}[showgrid=true](-1,-1)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\psset{Radius=\pstDistVal{2.0}}
\pstGeonode[PosAngle=0](1.5,1.5){O}
\pstCircleOA[linecolor=red]{O}{}
\pstCircleRotNode[PosAngle=0,RotAngle=0]{O}{}{A}
\pstCircleRotNode[PosAngle=60]{O}{}{B} % default 60 degree
\pstCircleRotNode[PosAngle=90,RotAngle=90]{O}{}{C}
\pstCircleRotNode[PosAngle=150,RotAngle=\pscalculate{3*360/7}]{O
    }{}{D}
\pstCircleRotNode[PosAngle=180,RotAngle=180]{O}{}{E}
\pstCircleRotNode[PosAngle=230,RotAngle=230]{O}{}{F}
\pstCircleRotNode[PosAngle=270,RotAngle=270]{O}{}{G}
\pstCircleNode[PosAngle=-45]{O}{}{-45}{H}
\end{pspicture}
```

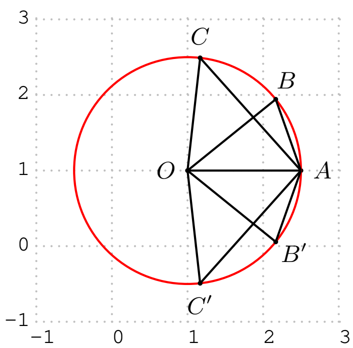
Sometimes we need to draw a chord with the given length from the start node, it is not possible to get the end node via the already defined macros, so we provide the macro `\pstCircleChordNode` to do this work. This macro find the node X on the circle such that the length of chord AX is the given value L, which can be got from `\pstDist`, `\pstDistConst`, `\pstDistAdd`, `\pstDistSub`, etc.

```
\pstCircleChordNode [Options] {O}{A}{L}{X}
```

The circle is just defined by center O and point A in this macro, so you can't omit the parameter A.

The direction to find node X is anti-clockwise by default. The parameter `CurvAbsNeg`(by default false) can change this behavior.

At last, the chord length L shouldn't large than the diameter of the circle, else we will put the node X at origin.

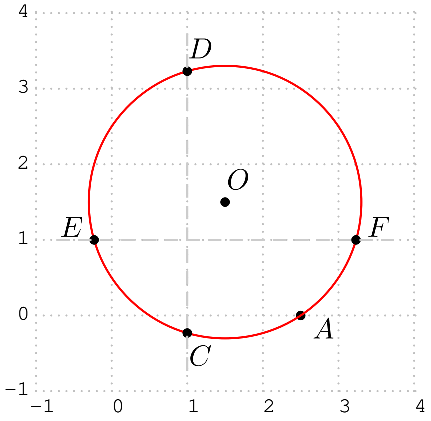



```
\begin{pspicture}[showgrid=true](-1,-1)(3,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\pstGeonode[PosAngle={180,0}](1,1){O}(2.5,1){A}
\pstCircleOA[linecolor=red]{O}{A}
\pstCircleChordNode[PosAngle=60]{O}{A}{\pstDistConst{1}}{B}
\pstCircleChordNode[PosAngle=90]{O}{A}{\pstDistConst{2}}{C}
\pstCircleChordNode[PosAngle=-30,CurvAbsNeg=true]{O}{A}{\
    pstDistConst{1}}{B'}
\pstCircleChordNode[PosAngle=-90,CurvAbsNeg=true]{O}{A}{\
    pstDistConst{2}}{C'}
\pstLineAB{O}{A}\pstLineAB{O}{B}\pstLineAB{O}{C}
\pstLineAB{O}{B'}\pstLineAB{O}{C'}
\pstLineAB{A}{B}\pstLineAB{A}{C}
\pstLineAB{A}{B'}\pstLineAB{A}{C'}
\end{pspicture}
```

A point can be positioned on a circle using its absolute abscissa or ordinate too. You can input x_1 or y_1 as any number(e.g, 2.0), or use `\pscalculate` or `\fpeval` to generate the value, or use `\pstAbscissa` and `\pstOrdinate` to get the abscissa and ordinate of any other node.

```
\pstCircleAbsNode [Options] {O}{A}{x1}{C}{D}
\pstCircleOrdNode [Options] {O}{A}{y1}{C}{D}
```

for example,



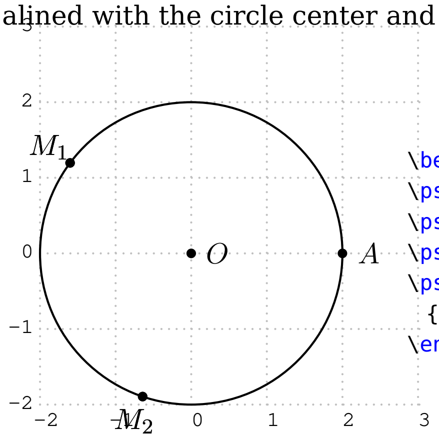

```
\begin{pspicture}[showgrid=true](-1,-1)(4,4)
\pstGeonode[PosAngle=60](1.5,1.5){O}
\pstGeonode[PosAngle=-30](2.5,0){A}
\pstCircleOA[linecolor=red]{O}{A}
\pstCircleAbsNode[PosAngle={-60,60},PointSymbol=*]{O}{A}{1.0}{C}{
    D}
\pstCircleOrdNode[PosAngle={150,30},PointSymbol=*]{O}{A}{1.0}{E}{
    F}
\pstLineAB[linestyle=dashed,linecolor=gray!40,nodesep=-0.5]{C}{D}
\pstLineAB[linestyle=dashed,linecolor=gray!40,nodesep=-0.5]{E}{F}
\end{pspicture}
```

A point can be positioned on a circle using its curved abscissa, that is, the arc length from a given node.

`\pstCurvAbsNode [Options]` $\{O\}\{A\}\{B\}\{Abs\}$

Possible optional arguments are `PointSymbol`, `PosAngle`, `PointName`, `PointNameSep`, `PtNameMath`, and `CurvAbsNeg`. The point $\langle B \rangle$ is positioned on the circle of center $\langle O \rangle$ crossing $\langle A \rangle$, with the curved abscissa ⟨Abs⟩. The origin is $\langle A \rangle$ and the direction is anti-clockwise by default. The parameter `CurvAbsNeg` (by default false) can change this behavior.

If the parameter `PosAngle` is not specified, the point label is put automatically in oirder to be alined with the circle center and the point.

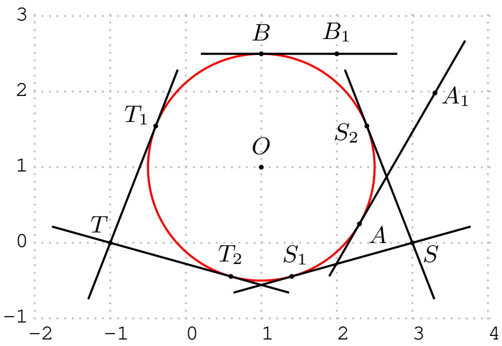



```
\begin{pspicture}[showgrid](-2.5,-2.5)(2.5,2.5)
\pstGeonode{O}(2,0){A}
\pstCircleOA{O}{A}
\pstCurvAbsNode{O}{A}{M_1}{\pstDistVal{5}}
\pstCurvAbsNode[CurvAbsNeg=true]%
 {O}{A}{M_2}{\pstDistAB{A}{M_1}}
\end{pspicture}
```

2.11. Circle tangent

The macro `\pstCircleTangentLine` is used to draw a tangent line AT from a point A on the circle, and the macro `\pstCircleTangentNode` is used to draw the tangent points T_1 and T_2 from a point P out of the circle.

```
\pstCircleTangentLine [Options] {O}{A}{T}
\pstCircleTangentNode [Options] {O}{A}{P}{T1}{T2}
```

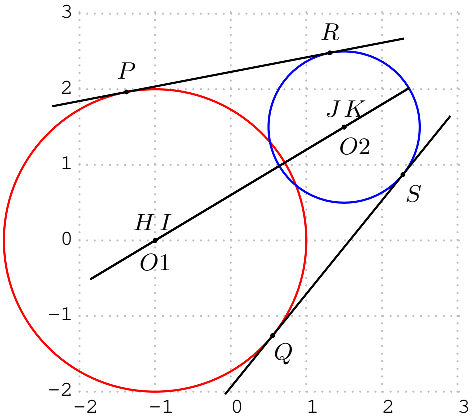



```
\begin{pspicture}[showgrid=true](-2,-1)(4,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\psset{nodesep=-0.8}
\pstGeonode[PosAngle={90,120,-30}](1,1){O}(-1,0){T}(3,0){S}
\pstCircleOA[Radius=\pstDistVal{1.5},linecolor=red]{O}{}
\pstCircleRotNode[Radius=\pstDistVal{1.5},PosAngle=-30,RotAngle
    =-30]{O}{}{A}
\pstCircleTangentLine[PosAngle=-10,PointName=A_1]{O}{A}{A1}
\pstCircleRotNode[Radius=\pstDistVal{1.5},PosAngle=90,RotAngle
    =90]{O}{}{B}
\pstCircleTangentLine[PosAngle=90,PointName=B_1]{O}{B}{B1}
\pstCircleTangentNode[Radius=\pstDistVal{1.5},PosAngle={150,90},
    PointName={T_1,T_2}]{O}{}{T}{T1}{T2}
\pstCircleTangentNode[PosAngle={80,200},PointName={S_1,S_2}]{O}{A
    }{S}{S1}{S2}
\end{pspicture}
```

The macro `\pstCircleExternalCommonTangent` is used to find the external common tangent lines of two circle $A(O_1)$ and $B(O_2)$, and the macro `\pstCircleInternalCommonTangent` is used to find the internal common tangent lines of two circle $A(O_1)$ and $B(O_2)$. They both create four tangent point nodes T_1, T_2, T_3, T_4, where T_1, T_2 lie on circle $A(O_1)$, and T_3, T_4 lie on circle $B(O_2)$.

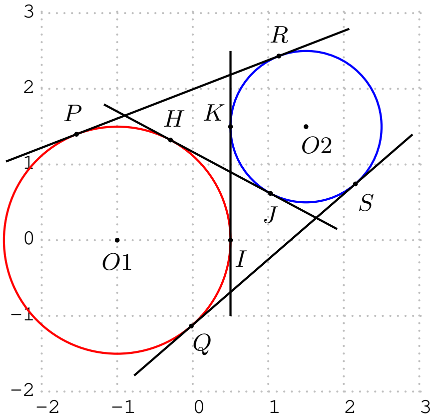
`\pstCircleExternalCommonTangent [Options]` $\{O_1\}\{A\}\{O_2\}\{B\}\{T_1\}\{T_2\}\{T_3\}\{T_4\}$
`\pstCircleInternalCommonTangent [Options]` $\{O_1\}\{A\}\{O_2\}\{B\}\{T_1\}\{T_2\}\{T_3\}\{T_4\}$

You can use `RadiusA` and `RadiusB` to define the two circles like as following:




```
\begin{pspicture}[showgrid=true](-2,-2)(3,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\pstGeonode[PosAngle=-90](-1,0){O1}
\pstGeonode[PosAngle=-60](1.5,1.5){O2}
\pstCircleOA[Radius=\pstDistVal{2},linecolor=red]{O1}{}
\pstCircleOA[Radius=\pstDistVal{1},linecolor=blue]{O2}{}
\pstCircleExternalCommonTangent[RadiusA=\pstDistVal{2},RadiusB=\
    pstDistVal{1},PosAngle={90,-60,90,-60}]{O1}{}{O2}{}{P}{Q}{R
    }{S}
\pstLine[nodesep=-1]{P}{R}
\pstLine[nodesep=-1]{Q}{S}
\pstCircleInternalCommonTangent[RadiusA=\pstDistVal{2},RadiusB=\
    pstDistVal{1},PosAngle={120,60,120,60}]{O1}{}{O2}{}{H}{I}{J
    }{K}
\pstLine[nodesep=-1]{H}{J}
\pstLine[nodesep=-1]{I}{K}
\end{pspicture}
```

You also can use `DiameterA` and `DiameterB` to define the two circles like as following:




```
\begin{pspicture}[showgrid=true](-2,-2)(3,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\pstGeonode[PosAngle=-90](-1,0){O1}
\pstGeonode[PosAngle=-60](1.5,1.5){O2}
\pstCircleOA[Diameter=\pstDistVal{3},linecolor=red]{O1}{}
\pstCircleOA[Diameter=\pstDistVal{2},linecolor=blue]{O2}{}
\pstCircleExternalCommonTangent[DiameterA=\pstDistVal{3},
    DiameterB=\pstDistVal{2},PosAngle={100,-60,90,-60}]{O1}{}{O
    2}{}{P}{Q}{R}{S}
\pstLine[nodesep=-1]{P}{R}
\pstLine[nodesep=-1]{Q}{S}
\pstCircleInternalCommonTangent[DiameterA=\pstDistVal{3},
    DiameterB=\pstDistVal{2},PosAngle={80,-60,-90,140}]{O1}{}{O
    2}{}{H}{I}{J}{K}
\pstLine[nodesep=-1]{H}{J}
\pstLine[nodesep=-1]{I}{K}
\end{pspicture}
```

2.12. Circle radical axis

If you want to draw the `Radical Axis` of two given circles, read the following sentenses. For given $\odot O_1$ with radius r_1 and $\odot O_2$ with radius r_2, and the center $O_1(x_1, y_1)$, $O_2(x_2, y_2)$, then any point $P(x, y)$ on the `Radical Axis` is satisfied:

$$(x - x_1)^2 + (y - y_1)^2 - r_1^2 = (x - x_2)^2 + (y - y_2)^2 - r_2^2$$

It can be simplified to a equation of a line:

$$2(x_2 - x_1)x + 2(y_2 - y_1)y = (x_2^2 + y_2^2 - r_2^2) - (x_1^2 + y_1^2 - r_1^2)$$

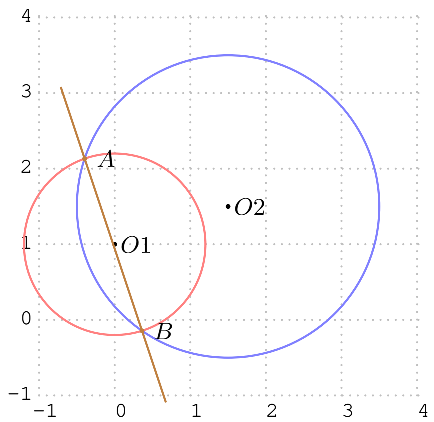
It is clear that the circles with same center have no radical axis, and the radical axis is perpendicular to the line of centers.

We provide the macro `\pstCircleRadicalAxis` to draw the `Radical Axis` of two given circles. It can handler every position relations of circles such as separation, intersection and inclusion.

`\pstCircleRadicalAxis [Options]` $\{O_1\}\{A\}\{O_2\}\{B\}\{C\}\{D\}$

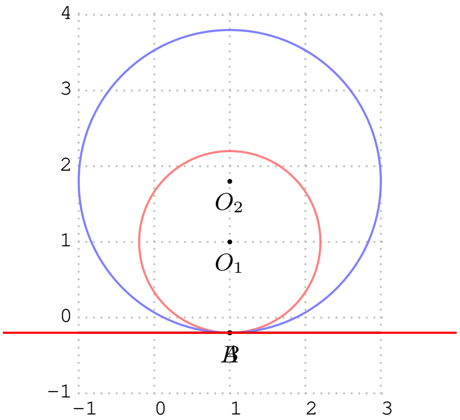
Both parameter A and B can be omitted and then to specify the each radius or diameter using the parameters `RadiusA`, `DiameterA`, and `RadiusB`, `DiameterB`. This macro create two new nodes C and D on the radical axis, you can find them in following examples.

When they are intersected, we can see the radical axis is the intersected chord line.



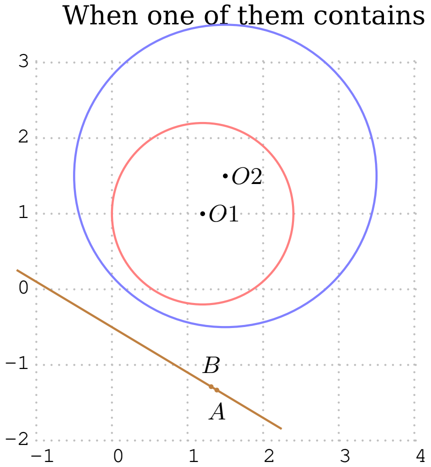

```
\begin{pspicture}[showgrid=true](-1,-1)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\ra{1.2}\def\rb{2.0}
\pstGeonode[PosAngle=0](0,1){O1}(1.5,1.5){O2}
\pstCircleOA[linecolor=red!50,Radius=\pstDistVal{\ra}]{O1}{}
\pstCircleOA[linecolor=blue!50,Radius=\pstDistVal{\rb}]{O2}{}
\pstCircleRadicalAxis[PosAngle={0,0},RadiusA=\pstDistVal{\ra},
    RadiusB=\pstDistVal{\rb},nodesep=-1,linecolor=brown]{O1}{}{O
    2}{}{A}{B}
\end{pspicture}
```

When they are tangent, we can see the radical axis is the common tangent line.



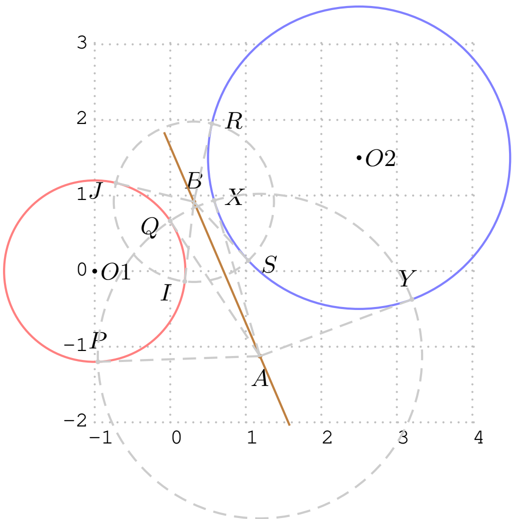

```
\begin{pspicture}[showgrid=true](-1,-1)(3,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\ra{1.2}\def\rb{2.0}
\pstGeonode[PosAngle=-90,PointName={O_1,O_2}](1,1){O1}(1,1.8){O2}
\pstCircleOA[linecolor=red!50,Radius=\pstDistVal{\ra}]{O1}{}
\pstCircleOA[linecolor=blue!50,Radius=\pstDistVal{\rb}]{O2}{}
\pstCircleRadicalAxis[nodesep=-2,PosAngle={-90,-90},RadiusA=\
    pstDistVal{\ra},RadiusB=\pstDistVal{\rb}]{O1}{}{O2}{}{A}{B}
\pstLineAB[linecolor=red,nodesep=-3]{A}{B}
\end{pspicture}
```

When one of them contains the other, the radical axis is out of the circles.




```
\begin{pspicture}[showgrid=true](-1,-2)(4,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\ra{1.2}\def\rb{2.0}
\pstGeonode[PosAngle=0](1.2,1){O1}(1.5,1.5){O2}
\pstCircleOA[linecolor=red!50,Radius=\pstDistVal{\ra}]{O1}{}
\pstCircleOA[linecolor=blue!50,Radius=\pstDistVal{\rb}]{O2}{}
\pstCircleRadicalAxis[PosAngle={-90,90},RadiusA=\pstDistVal{\ra},
    RadiusB=\pstDistVal{\rb},nodesepA=-1,nodesepB=-3,linecolor=
    brown]{O1}{}{O2}{}{A}{B}
\end{pspicture}
```

When they are separated, the radical axis is between of the circles.




```
\begin{pspicture}[showgrid=true](-1,-2)(4,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\ra{1.2}\def\rb{2.0}
\pstGeonode[PosAngle=0](-1,0){O1}(2.5,1.5){O2}
\pstCircleOA[linecolor=red!50,Radius=\pstDistVal{\ra}]{O1}{}
\pstCircleOA[linecolor=blue!50,Radius=\pstDistVal{\rb}]{O2}{}
\pstCircleRadicalAxis[PosAngle={-90,90},RadiusA=\pstDistVal{\ra},
    RadiusB=\pstDistVal{\rb},nodesep=-1,linecolor=brown]{O1}{}{O
    2}{}{A}{B}
\psset{linestyle=dashed,linecolor=gray!40}
\pstCircleTangentNode[Radius=\pstDistVal{\ra},PosAngle={90,200}]{
    O1}{}{A}{P}{Q}
\pstCircleTangentNode[Radius=\pstDistVal{\rb},PosAngle={10,100}]{
    O2}{}{A}{X}{Y}
\pstCircleOA{A}{P}
\pstCircleTangentNode[Radius=\pstDistVal{\ra},PosAngle
    ={210,200}]{O1}{}{B}{I}{J}
\pstCircleTangentNode[Radius=\pstDistVal{\rb},PosAngle={10,-10}]{
    O2}{}{B}{R}{S}
\pstCircleOA{B}{I}
\end{pspicture}
```

2.13. Regular polygons

For the 3-side and 4-side regular polygon, we provide the macro `\pstETriangleAB` and `\pstSquareAB` to draw them. In general, you can use the macro `\pstRegularPolygonAB` and `\pstRegularPolygonOA` to get a n-side regular polygon.

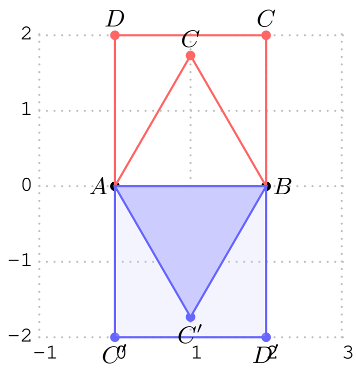
```
\pstETriangleAB [Options] {A}{B}{C}
\pstSquareAB [Options] {A}{B}{C}{D}
\pstRegularPolygonAB [Options] {A_0}{A_1}{n}{A_2,A_3,...,A_{n-1}}
\pstRegularPolygonOA [Options] {O}{A_0}{n}{A_1,A_2,...,A_{n-1}}
```

The macro `\pstETriangleAB` draw a equilateral triangle on a given side AB, and output the third node C; The macro `\pstSquareAB` draw a square on a given side AB, and output the other two nodes C, D; The macro `\pstRegularPolygonAB` draw a n-side regular polygon on a given side A_0A_1, and output the other nodes $A_2, A_3, \cdots, A_{n-1}$; The macro `\pstRegularPolygonOA` draw a n-side regular polygon with center O and base point A_0, and output the other nodes $A_1, A_2, \cdots, A_{n-1}$.

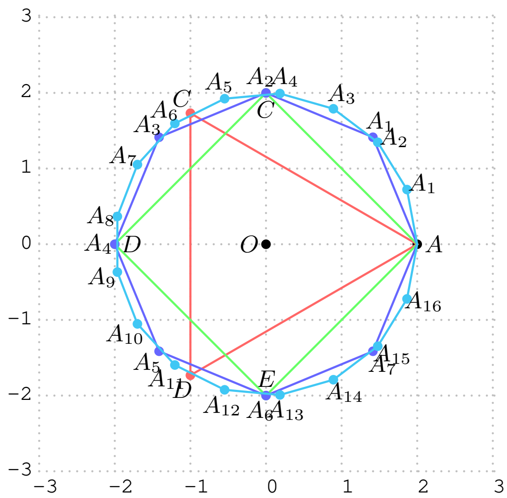
You can use the parameters `linestyle`, `linecolor`, `linewidth` to control the line style; and use the parameters `PointName`, `PosAngle`, `PointSymbol` to control the point nodes; and use the parameters `CurveType`, `fillstyle`, `fillcolor` to control the polygon style.

For the last output point list in macro `\pstRegularPolygonAB` and `\pstRegularPolygonOA`, if you do not enter a complete point list, the remaining points will be automatically named.

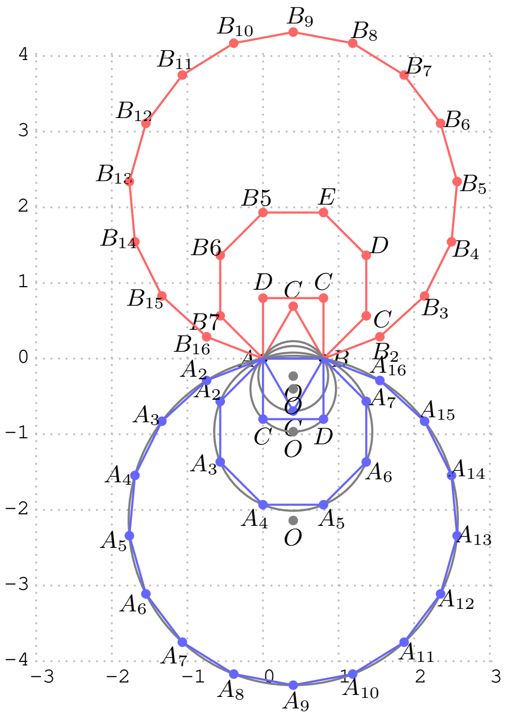
Note that draw regular polygon with side from A to B and side from B to A will get the figure symmetric to AB. Here are some examples.



```
\begin{pspicture}[showgrid](-1,-2)(3,2)
\psset{unit=0.40cm}\footnotesize\psset{PointSymbol=none,
    PointNameSep=0.22cm}
\pstGeonode[PosAngle={180,0},PointSymbol=*](0,0){A}(5,0){B}
\pstSquareAB[linecolor=red!60,PosAngle={90,90},PointSymbol=*]{A}{
    B}{C}{D}
\pstSquareAB[linecolor=blue!60,PosAngle={-90,-90},PointSymbol=*,
    fillstyle=solid,fillcolor=blue!20,opacity=0.2]{B}{A}{C'}{D'}
\pstETriangleAB[linecolor=red!60,PosAngle=90,PointSymbol=*]{A}{B
    }{C}
\pstETriangleAB[linecolor=blue!60,PosAngle=-90,PointSymbol=*,
    fillstyle=solid,fillcolor=blue!20]{B}{A}{C'}
\end{pspicture}
```



```
\begin{pspicture}[showgrid](-3,-3)(3,3)
\psset{unit=0.40cm}\footnotesize\psset{PointSymbol=none,
    PointNameSep=0.22cm}
\pstGeonode[PosAngle={180,0},PointSymbol=*](0,0){O}(5,0){A}
\pstRegularPolygonOA[CurveType=polygon,linecolor=red!60,
    PointSymbol=*,PosAngle={120,240}]{O}{A}{3}{C,D}
\pstRegularPolygonOA[CurveType=polygon,linecolor=green!60,
    PointSymbol=*,PosAngle={-90,0,90}]{O}{A}{4}{C,D,E}
\pstRegularPolygonOA[CurveType=polygon,linecolor=blue!60,
    PointSymbol=*,PosAngle={65,110,135,180,225,250,305},
    PointName={A_1,A_2,A_3,A_4,A_5,A_6,A_7}]{O}{A}{8}{A1,A2,A3,A
    4,A5,A6,A7}
\pstRegularPolygonOA[CurveType=polygon,linecolor=cyan!60,
    PointSymbol=*,PosAngle
    ={20,15,60,75,110,130,150,180,200,220,240,260,290,310,330,350},
    PointName={A_1,A_2,A_3,A_4,A_5,A_6,A_7,A_8,A_9,A_{10},A
    _{11},A_{12},A_{13},A_{14},A_{15},A_{16}}]{O}{A}{17}{A1,A2,A
    3,A4,A5,A6,A7,A8,A9,A10,A11,A12,A13,A14,A15,A16}
\end{pspicture}
```



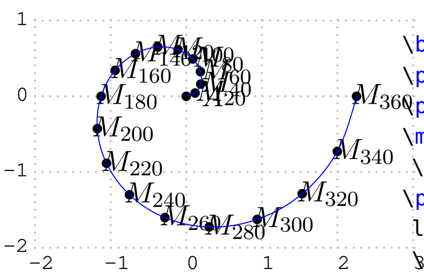
```
\begin{pspicture}[showgrid](-3,-4)(3,4)
\psset{unit=0.40cm}\footnotesize\psset{PointSymbol=none,
    PointNameSep=0.22cm}
\pstGeonode[PosAngle={180,0},PointSymbol=*](0,0){A}(2,0){B}
\pstRegularPolygonAB[CurveType=polygon,linecolor=red!60,
    PointSymbol=*,PosAngle={90,90,90}]{A}{B}{3}{C,D,E}
\pstRegularPolygonAB[CurveType=polygon,linecolor=red!60,
    PointSymbol=*,PosAngle={90,90}]{A}{B}{4}{C,D,E}
\pstRegularPolygonAB[CurveType=polygon,linecolor=red!60,
    PointSymbol=*,PosAngle={-10,40,80,115,150,200}]{A}{B}{8}{C,D
    ,E}
\pstRegularPolygonAB[CurveType=polygon,linecolor=red!60,
    PointSymbol=*,PosAngle
    ={290,310,330,350,10,30,45,65,85,115,135,155,170,190,210},
    PointName={B_2,B_3,B_4,B_5,B_6,B_7,B_8,B_9,B_{10},B_{11},B
    _{12},B_{13},B_{14},B_{15},B_{16}}]{A}{B}{17}{B2}
\psset{CodeFig=true,CodeFigColor=gray,CodeFigStyle=solid}
\pstRegularPolygonAB[CurveType=polygon,linecolor=blue!60,
    PointSymbol=*,PosAngle={-90,-90}]{B}{A}{3}{C,D,E}
\pstRegularPolygonAB[CurveType=polygon,linecolor=blue!60,
    PointSymbol=*,PosAngle={-90,-90}]{B}{A}{4}{C,D,E}
\pstRegularPolygonAB[CurveType=polygon,linecolor=blue!60,
    PointSymbol=*,PosAngle={140,190,240,-60,-40,10},PointName={A
    _2,A_3,A_4,A_5,A_6,A_7}]{B}{A}{8}{A2,A3,A4,A5,A6,A7}
\pstRegularPolygonAB[CurveType=polygon,linecolor=blue!60,
    PointSymbol=*,PosAngle
    ={140,160,180,200,220,240,260,280,300,320,340,360,20,40,60},
    PointName={A_2,A_3,A_4,A_5,A_6,A_7,A_8,A_9,A_{10},A_{11},A
    _{12},A_{13},A_{14},A_{15},A_{16}}]{B}{A}{17}{A2,A3,A4,A5,A
    6,A7,A8,A9,A10,A11,A12,A13,A14,A15,A16}
\end{pspicture}
```

2.14. Generic curve

It is possible to generate a set of points using a loop, and to give them a generic name defined by a radical and a number. The following command can draw a interpolated curve crossing all such kind of points.

`\pstGenericCurve [Options] {`*Radical*`}{`n_1`}{`n_2`}`

Possible optional arguments are `GenCurvFirst`, `GenCurvInc`, and `GenCurvLast` The curve is drawn on the points whose name is defined using the radical ⟨Radical⟩ followed by a number from ⟨n_1⟩ to ⟨n_2⟩. In order to manage side effect, the parameters `GenCurvFirst` et `GenCurvLast` can be used to specified special first or last point. The parameter `GenCurvInc` can be used to modify the increment from a point to the next one (by default 1).



```
\begin{pspicture}[showgrid](-2.5,-2.5)(2.5,1)
\psset{unit=.00625}
\pstGeonode{A}
\multido{\n=20+20}{18}{%
\pstGeonode[PointName=M_{\n}](\n;\n){M_\n}}
\pstGenericCurve[GenCurvFirst=A,GenCurvInc=20,
linecolor=blue,linewidth=.5\pslinewidth]{M_}{20}{360}
\end{pspicture}
```

3. Conics

3.1. Standard Ellipse

The Standard Ellipse E with coordinate translation is defined by center $O(x_0, y_0)$, the half of the major axis $max(abs(a), abs(b))$, the half of the minor axis $min(abs(a), abs(b))$, the equation as following:

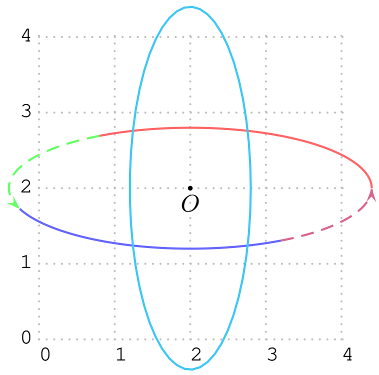
$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1 \tag{1}$$

Sometimes we use the parametric function of the Standard Ellipse with coordinate translation:

$$\begin{cases} x = a\cos\alpha + x_0 \\ y = b\sin\alpha + y_0 \end{cases} \tag{2}$$

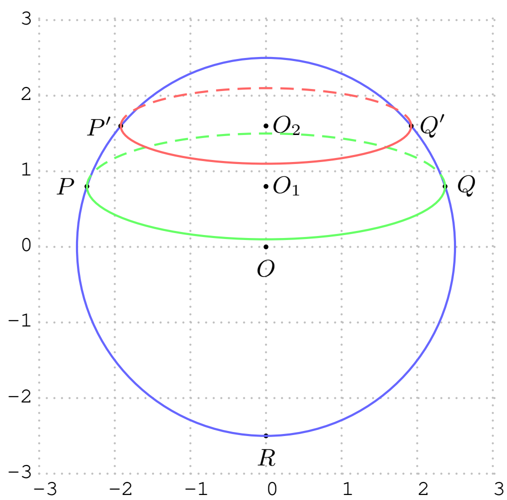
The Macro `\pstEllipse` is used to draw a Standard Ellipse with center O from `angleA` to `angleB`, going counter clockwise. It combines the function like `\psellipse` and `\psellipticarc` in `PSTricks`. If `angleA` and `angleB` are not specified, the macro will draw the whole ellipse.

`\pstEllipse [Options] (`O`)(`a, b`) [angleA] [angleB]`




```
\begin{pspicture}[showgrid=true](0,0)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\ra{2.4}\def\rb{0.8}
\pstGeonode[PosAngle=-90,PointNameSep=0.2](2,2){O}
%\psellipse[linecolor=red!60](O)(\ra,\rb)
\pstEllipse[linecolor=red!60](O)(\ra,\rb)[0][120]
\pstEllipse[linecolor=green!60,linestyle=dashed,arrows=->,
    arrowscale=1.2](O)(\ra,\rb)[120][200]
\pstEllipse[linecolor=blue!60](O)(\ra,\rb)[200][300]
\pstEllipse[linecolor=purple!60,linestyle=dashed,arrows=->,
    arrowscale=1.2](O)(\ra,\rb)[300][360]
\pstEllipse[linecolor=cyan!60](O)(\rb,\ra)
\end{pspicture}
```

Like as the coordinates, the parameters a, b can be got by the raw PostScript commands too, where you can use the macros `\pstDist*`, for example,

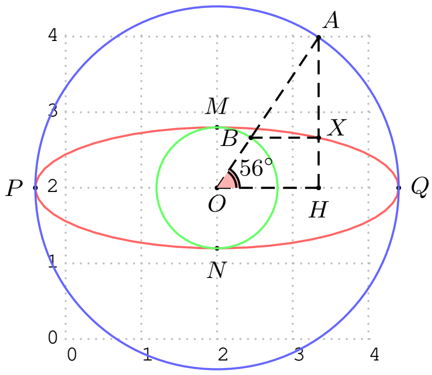



```
\begin{pspicture}[showgrid=true](-3,-3)(3,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\pstGeonode[PosAngle=-90,PointSymbol=*](0,0){O}(0,-2.5){R}
\pstCircleOA[linecolor=blue!60]{O}{R}
\pstGeonode[PosAngle=0,PointName=O_1,PointSymbol=*](0,0.8){O1}
\pstGeonode[PosAngle=0,PointName=O_2,PointSymbol=*](0,1.6){O2}
\pstCircleOrdNode[PointName={P,Q},PosAngle={180,0}]{O}{R}{\
    pstOrdinate{O1}}{P}{Q}
\pstCircleOrdNode[PointName={P',Q'},PosAngle={180,0}]{O}{R}{\
    pstOrdinate{O2}}{P'}{Q'}
\pstEllipse[linecolor=green!60,linestyle=dashed](O1)(! \
    pstUserDist{\pstDist{O1}{P}} 0.7)[0][180]
\pstEllipse[linecolor=green!60](O1)(! \pstUserDist{\pstDist{O1}{P
    }} 0.7)[180][360]
\pstEllipse[linecolor=red!60,linestyle=dashed](O2)(! \pstUserDist
    {\pstDist{O2}{P'}} 0.5)[0][180]
\pstEllipse[linecolor=red!60](O2)(! \pstUserDist{\pstDist{O2}{P
    '}} 0.5)[180][360]
\end{pspicture}
```

Now you can draw some points on this Ellipse using macro `\pstEllipseNode` or `\pstEllipseRotNode`. The macro `\pstEllipseNode` requires an explicit parameter t as α in equation (2) to calculate the point; but the macro `\pstEllipseRotNode` requires an implicit parameter `RotAngle` as α in equation (2) to calculate the point.

`\pstEllipseNode [Options]` $(O)(a,b)\{t\}\{P\}$
`\pstEllipseRotNode [Options]` $(O)(a,b)\{P\}$

The following is the example, note that the `RotAngle` is not $\angle HOX$ in geometrical, but $\angle HOA$ or $\angle HOB$.

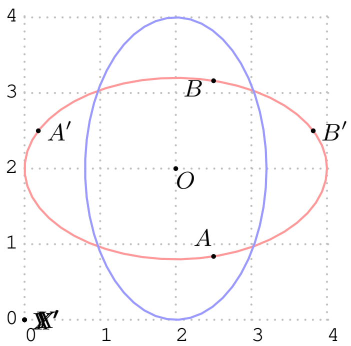



```
\begin{pspicture}[showgrid=true](0,0)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\ra{2.4}\def\rb{0.8}\def\rot{56}
\pstGeonode[PosAngle=-90,PointNameSep=0.2](2,2){O}
%\psellipse[linecolor=red!60](O)(\ra,\rb)
\pstEllipse[linecolor=red!60](O)(\ra,\rb)
\pstEllipseNode[PosAngle=180](O)(\ra,\rb){180}{P}
\pstEllipseRotNode[PosAngle=0,RotAngle=0](O)(\ra,\rb){Q}
\pstEllipseRotNode[PosAngle=90,RotAngle=90](O)(\ra,\rb){M}
\pstEllipseRotNode[PosAngle=-90,RotAngle=-90](O)(\ra,\rb){N}
\pstCircleOA[linecolor=blue!60,Radius=\pstDistVal{\ra}]{O}{}
\pstCircleRotNode[PosAngle=\rot,RotAngle=\rot,Radius=\pstDistVal
    {\ra}]{O}{}{A}
\pstCircleOA[linecolor=green!60,Radius=\pstDistVal{\rb}]{O}{}
\pstCircleRotNode[PosAngle=180,RotAngle=\rot,Radius=\pstDistVal{\
    rb}]{O}{}{B}
\pstEllipseRotNode[PosAngle=30,RotAngle=\rot](O)(\ra,\rb){X}
\pstProjection[PosAngle=-90]{P}{Q}{A}[H]
\pstLineAB[linestyle=dashed]{A}{O}
\pstLineAB[linestyle=dashed]{A}{H}
\pstLineAB[linestyle=dashed]{B}{X}
\pstLineAB[linestyle=dashed]{O}{H}
\pstMarkAngle[LabelSep=.6,MarkAngleRadius=.3,MarkAngleType=double
    ,fillcolor=red!30,fillstyle=solid]{H}{O}{A}{$\rot^\circ$}
\end{pspicture}
```

The macros `\pstEllipseAbsNode` and `\pstEllipseOrdNode` are used to get the two nodes A and B whose abscissas or ordinates are the given value x_1 or y_1 on the Standard Ellipse E.

If there is no such point satisfied this condition, then the nodes A and B will be put at the origin.

`\pstEllipseAbsNode [Options]` $(O)(a,b)\{x_1\}\{A\}\{B\}$
`\pstEllipseOrdNode [Options]` $(O)(a,b)\{y_1\}\{A\}\{B\}$

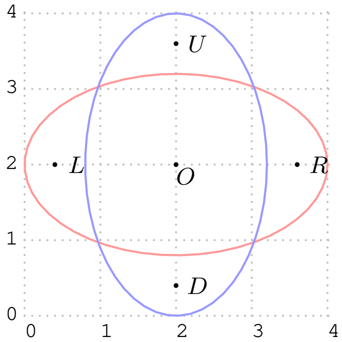



```
\begin{pspicture}[showgrid=true](0,0)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\ra{2.0}\def\rb{-1.2}
\pstGeonode[PosAngle=-50,PointNameSep=0.2](2,2){O}
\pstEllipse[linecolor=red!40](O)(\ra,\rb)
\pstEllipse[linecolor=blue!40](O)(\rb,\ra)
\pstEllipseAbsNode[PosAngle={120,200}](O)(\ra,\rb){2.5}{A}{B}
\pstEllipseAbsNode(O)(\ra,\rb){6}{X}{Y} % not exist
\pstEllipseOrdNode(O)(\ra,\rb){2.5}{A'}{B'}
\pstEllipseOrdNode(O)(\ra,\rb){6}{X'}{Y'} % not exist
\end{pspicture}
```

Here we find the focus node of Standard Ellipse! Please use macro `\pstEllipseFocusNode` to do this work.

`\pstEllipseFocusNode [Options] (`O`)(`a,b`){`A`}{`B`}`

For example:

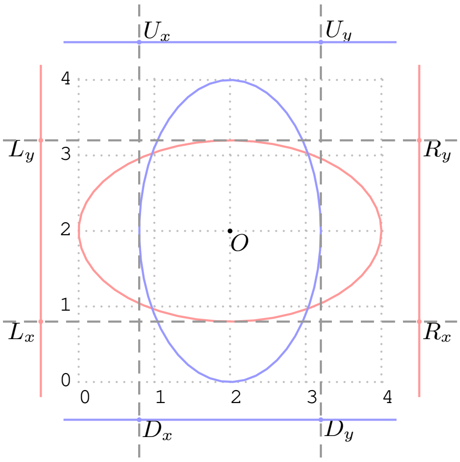



```
\begin{pspicture}[showgrid=true](0,0)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\ra{2.0}\def\rb{-1.2}
\pstGeonode[PosAngle=-50,PointNameSep=0.2](2,2){O}
\pstEllipse[linecolor=red!40](O)(\ra,\rb)
\pstEllipse[linecolor=blue!40](O)(\rb,\ra)
\pstEllipseFocusNode(O)(\ra,\rb){L}{R}
\pstEllipseFocusNode(O)(\rb,\ra){D}{U}
\end{pspicture}
```

The macro `\pstEllipseDirectrixLine` is used to draw the two directrix lines of Standard Ellipse, and create two new nodes on each of them. The nodes L_x, L_y are on the left/down directrix line, and R_x, R_y are on the right/up directrix line. They are lie on the tangent line of the vertex on the other axis.

`\pstEllipseDirectrixLine [Options] (`O`)(`a,b`){`L_x`}{`L_y`}{`R_x`}{`R_y`}`

For example:

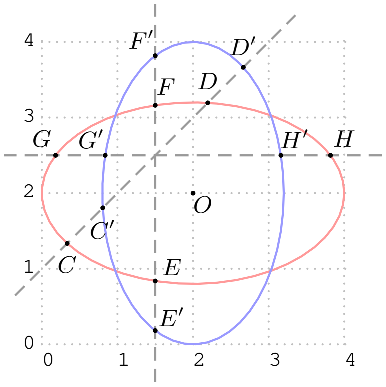



```
\begin{pspicture}[showgrid=true](0,0)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\ra{2.0}\def\rb{-1.2}
\pstGeonode[PosAngle=-50,PointNameSep=0.2](2,2){O}
\pstEllipse[linecolor=red!40](O)(\ra,\rb)
\pstEllipse[linecolor=blue!40](O)(\rb,\ra)
\pstEllipseDirectrixLine[PointName={L_x,L_y,R_x,R_y},PosAngle
    ={210,210,-30,-30},nodesep=-1,linecolor=red!40](O)(\ra,\rb){
    Lx}{Ly}{Rx}{Ry}
\pstEllipseDirectrixLine[PointName={D_x,D_y,U_x,U_y},PosAngle
    ={-30,-30,30,30},nodesep=-1,linecolor=blue!40](O)(\rb,\ra){
    Dx}{Dy}{Ux}{Uy}
\pstLine[nodesep=-0.5,linecolor=black!40,linestyle=dashed]{Lx}{Rx
    }
\pstLine[nodesep=-0.5,linecolor=black!40,linestyle=dashed]{Ly}{Ry
    }
\pstLine[nodesep=-0.5,linecolor=black!40,linestyle=dashed]{Dx}{Ux
    }
\pstLine[nodesep=-0.5,linecolor=black!40,linestyle=dashed]{Dy}{Uy
    }
\end{pspicture}
```

Sometimes we need to find the intersection of Ellipse and line, the Macro `\pstEllipseLineInter` can do this work, and it can handle any type of line, i.e, horizontal, vertical or others lines. It get the two intersection C and D of the Standard Ellipse E and the given line AB. When there is none intersection, C and D are both put at the origin; When there is only on intersection, it will be saved at node C, and D will be put at the origin.

`\pstEllipseLineInter [Options] (`O`)(`a,b`){`A`}{`B`}{`C`}{`D`}`

Here is examples:



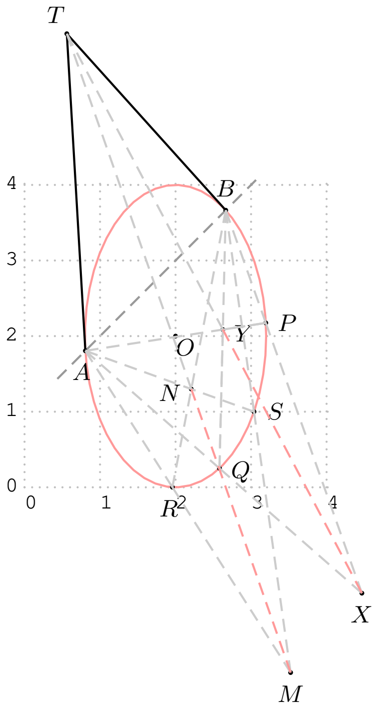

```
\begin{pspicture}[showgrid=true](0,0)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\ra{2.0}\def\rb{-1.2}
\pstGeonode[PosAngle=-50,PointNameSep=0.2](2,2){O}
\pstEllipse[linecolor=red!40](O)(\ra,\rb)
\pstEllipse[linecolor=blue!40](O)(\rb,\ra)
\pstLine[nodesep=-0.5,linecolor=black!40,linestyle=dashed
    ]{0,1}{3,4}
\pstEllipseLineInter[PosAngle={-90,90}](O)(\ra,\rb){0,1}{3,4}{C}{
    D}
\pstEllipseLineInter[PosAngle={-90,90}](O)(\rb,\ra){0,1}{3,4}{C
    '}{D'}
\pstLine[nodesep=-0.5,linecolor=black!40,linestyle=dashed
    ]{1.5,0}{1.5,4}
\pstEllipseLineInter[PosAngle={40,60}](O)(\ra,\rb){1.5,0}{1.5,4}{
    E}{F}
\pstEllipseLineInter[PosAngle={40,130}](O)(\rb,\ra)
    {1.5,1}{1.5,4}{E'}{F'}
\pstLine[nodesep=-0.5,linecolor=black!40,linestyle=dashed
    ]{4,2.5}{0,2.5}
\pstEllipseLineInter[PosAngle={130,50}](O)(\ra,\rb)
    {4,2.5}{0,2.5}{G}{H}
\pstEllipseLineInter[PosAngle={130,50}](O)(\rb,\ra)
    {4,2.5}{0,2.5}{G'}{H'}
\end{pspicture}
```

The macro `\pstEllipsePolarNode` is use to draw the tangent line of a point A or B on the Standard Ellipse. It draws the every tangent line through the point A and B on the Standard Ellipse E and get the insection node T of the two tangent lines. We call T as the polar point of chord AB as normal.

`\pstEllipsePolarNode [Options]` $(O)(a,b)\{A\}\{B\}\{T\}$

We use the following theorem to find the node T:

Theorem 1 *Give chord AB on the ellipse, we draw any other two chords PQ and RS, AB and PQ intersect at I, AQ and BP intersect at X, AP and BQ intersect at Y, we call XY is the polar line of point I. Also AB and RS intersect at J, AR and BS intersect at M, AS and BR intersect at N, we call MN is the polar line of point J. Then the intersection T of XY and MN is the polar point of chord AB, i.e. TA is the tangent line through A and TB is the tangent line through B.*



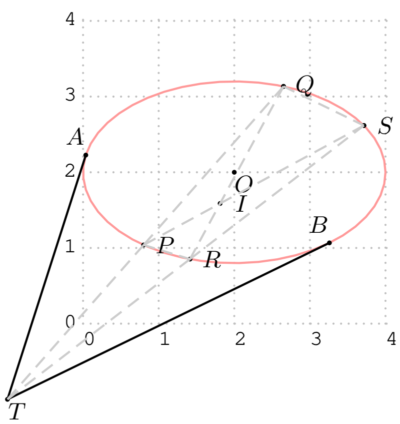

```
\begin{pspicture}[showgrid=true](0,0)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\rb{2.0}\def\ra{-1.2}
\pstGeonode[PosAngle=-50,PointNameSep=0.2](2,2){O}
\pstEllipse[linecolor=red!40](O)(\ra,\rb)
\pstLine[nodesep=-0.8,linecolor=black!40,linestyle=dashed
    ]{1,2}{2.5,3.5}
\pstEllipseLineInter[PosAngle={-100,90}](O)(\ra,\rb)
    {1,2}{2.5,3.5}{A}{B}
\pstEllipsePolarNode[PosAngle=120](O)(\ra,\rb){A}{B}{T}
% Here are the auxiliary lines to explain Theorem 1.
\pstEllipseRotNode[PosAngle=0,RotAngle=5](O)(\ra,\rb){P}
\pstEllipseRotNode[PosAngle=-10,RotAngle=-61](O)(\ra,\rb){Q}
\pstEllipseRotNode[PosAngle=-100,RotAngle=-92](O)(\ra,\rb){R}
\pstEllipseRotNode[PosAngle=0,RotAngle=-30](O)(\ra,\rb){S}
\pstInterLL[PosAngle=-90]{A}{Q}{B}{P}{X}
\pstInterLL[PosAngle=-10]{A}{P}{B}{Q}{Y}
\pstInterLL[PosAngle=-90]{A}{R}{B}{S}{M}
\pstInterLL[PosAngle=190]{A}{S}{B}{R}{N}
\psset{linestyle=dashed,linecolor=gray!40}
\pstLine{A}{Q}\pstLine{B}{P}\pstLine{A}{P}\pstLine{B}{Q}
\pstLine{A}{R}\pstLine{B}{S}\pstLine{A}{S}\pstLine{B}{R}
\pstLine{Q}{X}\pstLine{Q}{Y}\pstLine{P}{X}\pstLine{P}{Y}
\pstLine{R}{M}\pstLine{S}{M}\pstLine{T}{Y}\pstLine{T}{N}
\pstLine[linestyle=dashed,linecolor=red!40]{X}{Y}
\pstLine[linestyle=dashed,linecolor=red!40]{M}{N}
\end{pspicture}
```

The macro `\pstEllipseTangentNode` is use to draw the tangent line of a point T out of the Standard Ellipse E. It draw the two tangent lines through the point T to the Standard Ellipse E and get the node A and B on the Ellipse.

`\pstEllipseTangentNode [Options]` $(O)(a,b)\{T\}\{A\}\{B\}$

We use the following theorem to find the tangent node of the given T:

Theorem 2 *Give point T outside of the ellipse, we draw any other two chords TPQ and TRS, let PS and QR intersect at I, PR and QS intersect at X, XI and Ellipse intersect at A and B, then TA is the tangent line through A and TB is the tangent line through B.*




```
\begin{pspicture}[showgrid=true](0,0)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\ra{2.0}\def\rb{-1.2}
\pstGeonode[PosAngle=-50,PointNameSep=0.2](2,2){O}
\pstEllipse[linecolor=red!40](O)(\ra,\rb)
\pstGeonode[PosAngle=-50,PointNameSep=0.2](-1,-1){T}
\pstEllipseTangentNode[PosAngle=120](O)(\ra,\rb){T}{A}{B}
% Here are the auxiliary lines to explain Theorem 2.
\pstEllipseRotNode[PointName=none,RotAngle=71](O)(\ra,\rb){P0}
\pstEllipseRotNode[PointName=none,RotAngle=31](O)(\ra,\rb){R0}
\pstEllipseLineInter[PosAngle=0](O)(\ra,\rb){T}{P0}{P}{Q}
\pstEllipseLineInter[PosAngle=0](O)(\ra,\rb){T}{R0}{R}{S}
\pstInterLL[PosAngle=0]{P}{S}{Q}{R}{I}
\pstInterLL[PosAngle=0]{P}{R}{Q}{S}{X}
\psset{linestyle=dashed,linecolor=gray!40}
\pstLine{T}{P}\pstLine{P}{Q}\pstLine{T}{R}\pstLine{R}{S}
\pstLine{P}{S}\pstLine{Q}{R}\pstLine{P}{R}\pstLine{Q}{S}
\end{pspicture}
```

$\cdot X$

3.2. General Ellipse

Now we will introduce some macros for the General Ellipse as same as the Standard Ellipse. The General Ellipse E with coordinate translation and rotation is defined by center $O(x_0, y_0)$, the half of the major axis $max(abs(a), abs(b))$, the half of the minor axis $min(abs(a), abs(b))$, and the rotation angle θ of the major axis.

The equation can be got from the parametric function of the ellipse equation (2), using the rotation transform formula:

$$\begin{cases} x' = x\cos\theta - y\sin\theta \\ y' = x\sin\theta + y\cos\theta \end{cases} \tag{3}$$

then we have

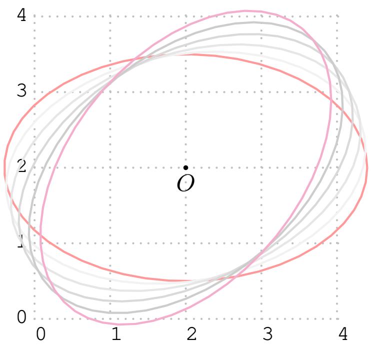
$$\begin{cases} x' = (a\cos\alpha + x_0)\cos\theta - (b\sin\alpha + y_0)\sin\theta = a\cos\alpha\cos\theta - b\sin\alpha\sin\theta + x_0' \\ y' = (a\cos\alpha + x_0)\sin\theta + (b\sin\alpha + y_0)\cos\theta = a\cos\alpha\sin\theta + b\sin\alpha\cos\theta + y_0' \end{cases} \tag{4}$$

where the x_0' and y_0' are the coordinate of the given center O after rotation. So we get the parametric function of the General Ellipse with coordinate translation and rotation as following:

$$\begin{cases} x = a\cos\alpha\cos\theta - b\sin\alpha\sin\theta + x_0 \\ y = a\cos\alpha\sin\theta + b\sin\alpha\cos\theta + y_0 \end{cases} \tag{5}$$

The Macro `\pstGeneralEllipse` is used to draw a General Ellipse with center O from `angleA` to `angleB`, going counter clockwise. If `angleA` and `angleB` are not specified, the macro will draw the whole ellipse. If you not input rotation angle θ, the default value is $0°$, at this time, the result of this macro is same as `\pstEllipse`. That is, `\pstGeneralEllipse` is more complex than `\pstEllipse`!

`\pstGeneralEllipse [Options] (`O`)(`a, b`) [`θ`] [angleA] [angleB]`

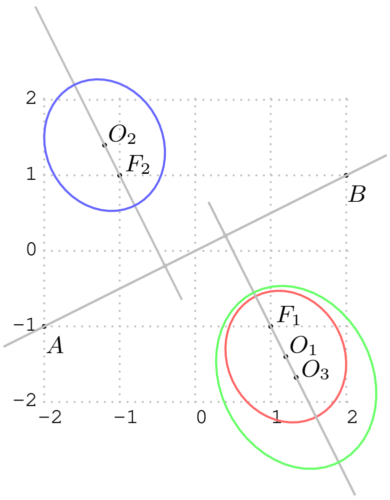



```
\begin{pspicture}[showgrid=true](0,0)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\ra{2.4}\def\rb{-1.5}
\pstGeonode[PosAngle=-90,PointNameSep=0.2](2,2){O}
\pstGeneralEllipse[linecolor=red!40](O)(\ra,\rb)[0]
\pstGeneralEllipse[linecolor=gray!10](O)(\ra,\rb)[10]
\pstGeneralEllipse[linecolor=gray!20](O)(\ra,\rb)[20]
\pstGeneralEllipse[linecolor=gray!30](O)(\ra,\rb)[30]
\pstGeneralEllipse[linecolor=gray!40](O)(\ra,\rb)[40]
\pstGeneralEllipse[linecolor=magenta!40](O)(\ra,\rb)[50]
\end{pspicture}
```

The Macro `\pstGeneralEllipseFle` is used to define a General Ellipse with Focus F, directrix line l, and the eccentricity e, where $0 \le e < 1$. It just calculate the center O, major radius a, minor radius b and the rotation angle θ of the major axis, then you can pass them into macro `\pstGeneralEllipse` to draw this ellipse.

`\pstGeneralEllipseFle [Options] `$\{F\}\{A\}\{B\}\{e\}\{O\}\{Rab\}\{\theta\}$

The output parameter `O` is a node name to store the center point, its label and symbol can be controlled by the options for `PSTricks` node, such as `PosAngle`. The output parameter `Rab` is a PostScript key to store the pair of major radius and minor radius, it just use `PSTricks` node coordinate to store a pair of value, but not a geometrical point. The output parameter θ is also a PostScript key to store the rotation angle of major axis, when you pass it to `\pstGeneralEllipse`, PostScript will lookup the value of this key in current dictionary.

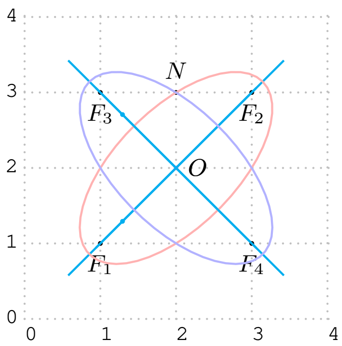



```
\begin{pspicture}[showgrid=true](-2,-2)(2,2)
\psset{unit=1.0cm}\psset{dotscale=0.5}\footnotesize
\psset{CodeFig=true,CodeFigColor=gray!50}\psset{PointSymbol=*}
\pstGeonode[PosAngle=30](1,-1){F_1}
\pstGeonode[PosAngle=30](-1,1){F_2}
\pstGeonode[PosAngle=-60](-2,-1){A}
\pstGeonode[PosAngle=-60](2,1){B}
\pstGeneralEllipseFle[PosAngle=30]{F_1}{A}{B}{0.5}{O_1}{R_1}{
    MajorRotAngle1}
\pstGeneralEllipse[linecolor=red!60](O_1)(R_1)[MajorRotAngle1]
\pstGeneralEllipseFle[PosAngle=30]{F_2}{A}{B}{0.5}{O_2}{R_2}{
    MajorRotAngle2}
\pstGeneralEllipse[linecolor=blue!60](O_2)(R_2)[MajorRotAngle2]
\pstGeneralEllipseFle[PosAngle=20]{F_1}{A}{B}{0.6}{O_3}{R_3}{
    MajorRotAngle3}
\pstGeneralEllipse[linecolor=green!60](O_3)(R_3)[MajorRotAngle3]
\end{pspicture}
```

The Macro `\pstGeneralEllipseFFN` is used to define a General Ellipse by the given focus nodes F_1, F_2, and one node N on it. It just calculate the center O, major radius a, minor radius b and the rotation angle θ of the major axis, then you can pass them into macro `\pstGeneralEllipse` to draw this ellipse.

`\pstGeneralEllipseFFN [Options]` $\{F_1\}\{F_2\}\{O\}\{Rab\}\{\theta\}$

The output parameter `O`, the output parameter `Rab` and the output parameter θ are same with `\pstGeneralEllipseFle`.

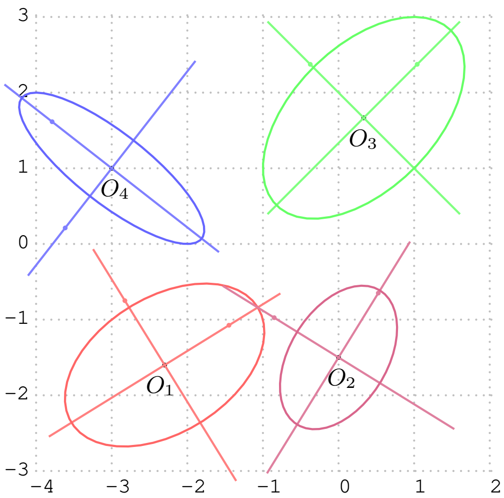



```
\begin{pspicture}[showgrid=true](0,0)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\pstGeonode[PosAngle=-90](1,1){F_1}
\pstGeonode[PosAngle=-90](3,3){F_2}
\pstGeonode[PosAngle=-90](1,3){F_3}
\pstGeonode[PosAngle=-90](3,1){F_4}
\pstGeonode[PosAngle=90](2,3){N}
\pstGeneralEllipseFFN[linecolor=red!30,CodeFig=true]{F_1}{F_2}{N
    }{O}{R1}{angle1}
\pstGeneralEllipse[linecolor=red!30](O)(R1)[angle1]
\pstGeneralEllipseFFN[linecolor=blue!30,CodeFig=true]{F_3}{F_4}{N
    }{O}{R2}{angle2}
\pstGeneralEllipse[linecolor=blue!30](O)(R2)[angle2]
\end{pspicture}
```

The Macro `\pstGeneralEllipseCoef` is used to define a General Ellipse by the quadratic curve equation $ax^2+bxy+cy^2+dx+ey+f=0$, it just calculate the center O, major radius a, minor radius b and the rotation angle θ of the major axis, then you can pass them into macro `\pstGeneralEllipse` to draw this ellipse. The package `pst-func` provides macro `\psplotImp` to draw an implicit defined functions too, but it can't tell you the geometrical elements like as center or radii, and it will take more time to calculate the function value point by point.

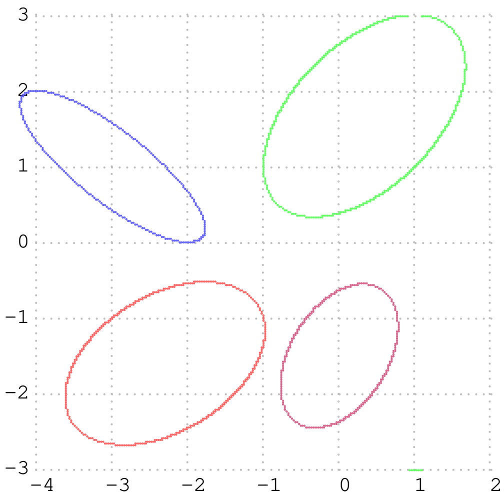
`\pstGeneralEllipseCoef [Options]` $\{a,b,c,d,e,f\}\{O\}\{Rab\}\{\theta\}$

The output parameter `O`, the output parameter `Rab` and the output parameter θ are same with `\pstGeneralEllipseFle`. They are set to zero if the coeffients are invalid to construct an ellipse.




```
\begin{pspicture}[showgrid=true](-4,-3)(2,3)
\psset{unit=1.0cm}\footnotesize\psset{dotscale=0.5}
\psset{CodeFig=true}\psset{PointSymbol=*}
%2x^2-2xy+3y^2+6x+5y+8=0
\pstGeneralEllipseCoef[PosAngle=-100,CodeFigColor=red
    !50]{2,-2,3,6,5,8}{O_1}{R_1}{MajorRotAngle1}
\pstGeneralEllipse[linecolor=red!60](O_1)(R_1)[MajorRotAngle1]
%3x^2-2xy+2y^2-3x+6y+3=0
\pstGeneralEllipseCoef[PosAngle=-80,CodeFigColor=purple
    !50]{3,-2,2,-3,6,3}{O_2}{R_2}{MajorRotAngle2}
\pstGeneralEllipse[linecolor=purple!60](O_2)(R_2)[MajorRotAngle2]
%x^2-xy+y^2+x-3y+1=0
\pstGeneralEllipseCoef[PosAngle=-90,CodeFigColor=green
    !50]{1,-1,1,1,-3,1}{O_3}{R_3}{MajorRotAngle3}
\pstGeneralEllipse[linecolor=green!60](O_3)(R_3)[MajorRotAngle3]
%2x^2+4xy+3y^2+8x+6y+8=0
\pstGeneralEllipseCoef[PosAngle=-80,CodeFigColor=blue
    !50]{2,4,3,8,6,8}{O_4}{R_4}{MajorRotAngle4}
\pstGeneralEllipse[linecolor=blue!60](O_4)(R_4)[MajorRotAngle4]
\end{pspicture}
```

You can verify the output figures with `\psplotImp` as following:

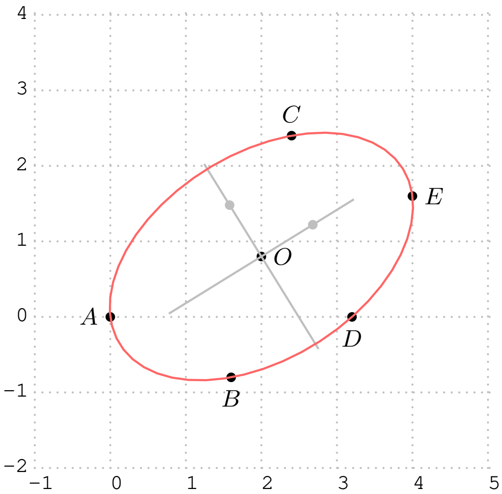



```
\begin{pspicture}[showgrid=true](-4,-3)(2,3)
\psset{unit=1.0cm}\footnotesize\psset{dotscale=0.5}
%2x^2-2xy+3y^2+6x+5y+8=0
\psplotImp[linecolor=red!60](-4,-3)(2,3){ 2 x dup mul mul -2 x
    mul y mul add 3 y dup mul mul add 6 x mul add 5 y mul add 8
    add }
%3x^2-2xy+2y^2-3x+6y+3=0
\psplotImp[linecolor=purple!60](-4,-3)(2,3){ 3 x dup mul mul -2 x
     mul y mul add 2 y dup mul mul add -3 x mul add 6 y mul add
    3 add }
%x^2-xy+y^2+x-3y+1=0
\psplotImp[linecolor=green!60](-4,-3)(2,3){ 1 x dup mul mul -1 x
    mul y mul add 1 y dup mul mul add 1 x mul add -3 y mul add 1
     add }
%2x^2+4xy+3y^2+8x+6y+8=0
\psplotImp[linecolor=blue!60](-5,-3)(2,3){ 2 x dup mul mul 4 x
    mul y mul add 3 y dup mul mul add 8 x mul add 6 y mul add 8
    add }
\end{pspicture}
```

The Macro `\pstGeneralEllipseABCDE` is used to define a General Ellipse by the given five points A, B, C, D, E, it just calculate the center O, major radius a, minor radius b and the rotation angle θ of the major axis, then you can pass them into macro `\pstGeneralEllipse` to draw this ellipse.

`\pstGeneralEllipseABCDE [Options]` $\{A\}\{B\}\{C\}\{D\}\{E\}\{O\}\{Rab\}\{\theta\}$

The output parameter `O`, the output parameter `Rab` and the output parameter θ are same with `\pstGeneralEllipseFle`. They are set to zero if the points are invalid to construct an ellipse.

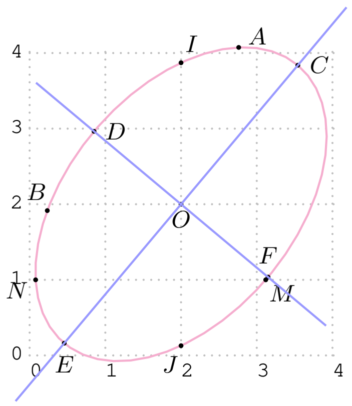



```
\begin{pspicture}[showgrid=true](-1,-2)(5,4)
\psset{unit=0.8cm}\footnotesize\psset{PointSymbol=*}
\psset{CodeFig=true,CodeFigColor=gray!50}
\pstGeonode[PosAngle=180](0,0){A}
\pstGeonode[PosAngle=-90](2,-1){B}
\pstGeonode[PosAngle=90](3,3){C}
\pstGeonode[PosAngle=-90](4,0){D}
\pstGeonode[PosAngle=0](5,2){E}
\pstGeneralEllipseABCDE[PosAngle=0]{A}{B}{C}{D}{E}{O}{R}{
    MajorRotAngle}
\pstGeneralEllipse[linecolor=red!60](O)(R)[MajorRotAngle]
\end{pspicture}
```

We can location the points on the General Ellipse using the macros `\pstGeneralEllipseNode`, `\pstGeneralEllipseRotNode`, `\pstGeneralEllipseAbsNode` and `\pstGeneralEllipseOrdNode` as following.

> `\pstGeneralEllipseNode` `[Options]` $(O)(a,\, b)$ `[`θ`]` $\{t\}\{A\}$
> `\pstGeneralEllipseRotNode` `[Options]` $(O)(a,\, b)$ `[`θ`]` $\{A\}$
> `\pstGeneralEllipseAbsNode` `[Options]` $(O)(a,\, b)$ `[`θ`]` $\{x_1\}\{A\}\{B\}$
> `\pstGeneralEllipseOrdNode` `[Options]` $(O)(a,\, b)$ `[`θ`]` $\{y_1\}\{A\}\{B\}$

Some examples all together:

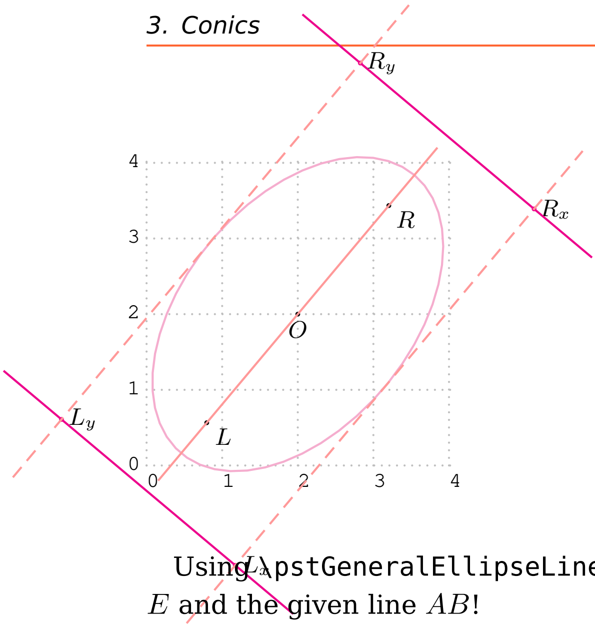



```
\begin{pspicture}[showgrid=true](0,0)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\ra{2.4}\def\rb{-1.5}
\pstGeonode[PosAngle=-90,PointNameSep=0.2](2,2){O}
\pstGeneralEllipse[linecolor=magenta!40](O)(\ra,\rb)[50]
\pstGeneralEllipseNode[PosAngle=30](O)(\ra,\rb)[50]{30}{A}
\pstGeneralEllipseRotNode[PosAngle=120,RotAngle=120](O)(\ra,\rb)
    [50]{B}
\pstGeneralEllipseRotNode[PosAngle=0,RotAngle=0](O)(\ra,\rb)[50]{
    C}
\pstGeneralEllipseRotNode[PosAngle=0,RotAngle=90](O)(\ra,\rb)
    [50]{D}
\pstGeneralEllipseRotNode[PosAngle=-90,RotAngle=180](O)(\ra,\rb)
    [50]{E}
\pstGeneralEllipseRotNode[PosAngle=90,RotAngle=-90](O)(\ra,\rb)
    [50]{F}
\pstGeneralEllipseAbsNode[PosAngle={60,240}](O)(\ra,\rb)[50]{2}{I
    }{J}
\pstGeneralEllipseOrdNode[PosAngle={-40,210}](O)(\ra,\rb)[50]{1}{
    M}{N}
\pstLineAB[nodesep=-1,linecolor=blue!40]{C}{E}
\pstLineAB[nodesep=-1,linecolor=blue!40]{D}{F}
\end{pspicture}
```

Using macro `\pstGeneralEllipseFocusNode` to find the two focus nodes, and macro `\pstGeneralEllipseDirectrixLine` to get the two directrix lines.

> `\pstGeneralEllipseFocusNode` `[Options]` $(O)(a,\, b)$ `[`θ`]` $\{t\}\{F_1\}\{F_2\}$
> `\pstGeneralEllipseDirectrixLine` `[Options]` $(O)(a,\, b)$ `[`θ`]` $\{L_x\}\{L_y\}\{R_x\}\{R_y\}$

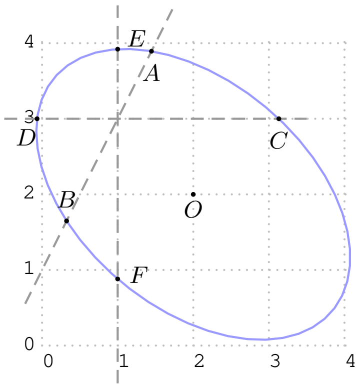
for example,




```
\begin{pspicture}[showgrid=true](0,0)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\ra{2.4}\def\rb{-1.5}
\pstGeonode[PosAngle=-90,PointNameSep=0.2](2,2){O}
\pstGeneralEllipse[linecolor=magenta!40](O)(\ra,\rb)[50]
\pstGeneralEllipseFocusNode[PosAngle={-40,-40}](O)(\ra,\rb)[50]{L
    }{R}
\pstGeneralEllipseDirectrixLine[PointName={L_x,L_y,R_x,R_y},
    nodesep=-1,linecolor=magenta](O)(\ra,\rb)[50]{Lx}{Ly}{Rx}{Ry
    }
\pstLine[nodesep=-1,linecolor=red!40]{L}{R}
\pstLine[nodesep=-1,linecolor=red!40,linestyle=dashed]{Lx}{Rx}
\pstLine[nodesep=-1,linecolor=red!40,linestyle=dashed]{Ly}{Ry}
\end{pspicture}
```

Using `\pstGeneralEllipseLineInter` to get the two intersections C and D of the General Ellipse E and the given line AB!

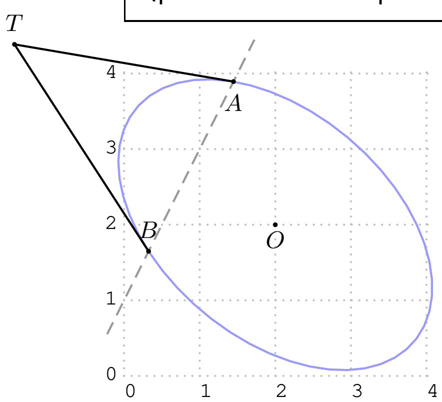
`\pstGeneralEllipseLineInter` `[Options]` $(O)(a,\, b)$ `[`θ`]` $\{A\}\{B\}\{C\}\{D\}$




```
\begin{pspicture}[showgrid=true](0,0)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\ra{1.5}\def\rb{-2.4}
\pstGeonode[PosAngle=-90,PointNameSep=0.2](2,2){O}
\pstGeneralEllipse[linecolor=blue!40](O)(\ra,\rb)[50]
\pstLine[nodesep=-0.5,linecolor=black!40,linestyle=dashed
    ]{0,1}{1.5,4}
\pstGeneralEllipseLineInter[PosAngle={-90,90}](O)(\ra,\rb)
    [50]{0,1}{1.5,4}{A}{B}
\pstLine[nodesep=-0.5,linecolor=black!40,linestyle=dashed
    ]{0,3}{3,3}
\pstGeneralEllipseLineInter[PosAngle={-90,240}](O)(\ra,\rb)
    [50]{0,3}{3,3}{C}{D}
\pstLine[nodesep=-0.5,linecolor=black!40,linestyle=dashed
    ]{1,0}{1,4}
\pstGeneralEllipseLineInter[PosAngle={30,10}](O)(\ra,\rb)
    [50]{1,1}{1,4}{E}{F}
\end{pspicture}
```

Using `\pstGeneralEllipsePolarNode` to find the polar point T of chord AB, please refer to Theorem 1.

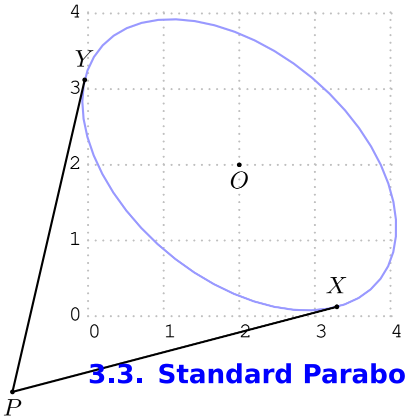
`\pstGeneralEllipsePolarNode` `[Options]` $(O)(a,\, b)$ `[`θ`]` $\{A\}\{B\}\{T\}$




```
\begin{pspicture}[showgrid=true](0,0)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\ra{1.5}\def\rb{-2.4}
\pstGeonode[PosAngle=-90,PointNameSep=0.2](2,2){O}
\pstGeneralEllipse[linecolor=blue!40](O)(\ra,\rb)[50]
\pstLine[nodesep=-0.5,linecolor=black!40,linestyle=dashed
    ]{0,1}{1.5,4}
\pstGeneralEllipseLineInter[PosAngle={-90,90}](O)(\ra,\rb)
    [50]{0,1}{1.5,4}{A}{B}
\pstGeneralEllipsePolarNode[PosAngle=90](O)(\ra,\rb)[50]{A}{B}{T}
\end{pspicture}
```

Using `\pstGeneralEllipseTangentNode` to find the tangent point A and B of outside point T, please refer to Theorem 2.

\pstGeneralEllipseTangentNode [Options] $(O)(a, b)$ [θ] $\{T\}\{A\}\{B\}$




```
\begin{pspicture}[showgrid=true](0,0)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\ra{1.5}\def\rb{-2.4}
\pstGeonode[PosAngle=-90,PointNameSep=0.2](2,2){O}
\pstGeneralEllipse[linecolor=blue!40](O)(\ra,\rb)[50]
\pstGeonode[PosAngle=-90,PointNameSep=0.2](-1,-1){P}
\pstGeneralEllipseTangentNode[PosAngle=90](O)(\ra,\rb)[50]{P}{X}{
    Y}
\end{pspicture}
```

3.3. Standard Parabola

The Standard Parabola P with coordinate translation is defined by vertex $O(x_0, y_0)$, the half of the focus chord axis $abs(p)$. Note that the sign of p indicates the direction of the parabola.

The equation can be written as:

$$(x - x_0)^2 = 2p(y - y_0) \tag{6}$$

and the parametric function can be written as:

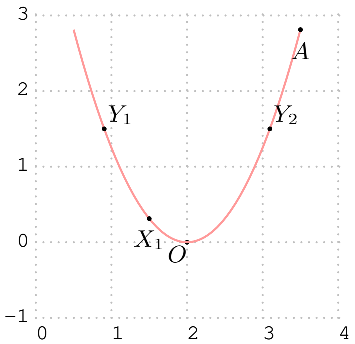
$$\begin{cases} x = t + x_0 \\ y = \dfrac{t^2}{2p} + y_0 \end{cases} \tag{7}$$

The macro \pstParabola is used to draw a Parabola from x_1 to x_2 with Vertex O, the half of the focus chord axis $abs(p)$.

\pstParabola [Options] $(O)\{p\}\{x_1\}\{x_2\}$

The macro \pstParabolaNode is used to draw a node whose parameter is the given value t on parabola, please refer to equation (7). The macro \pstParabolaAbsNode is used to draw a node whose abscissa is the given value x_1 on parabola. The macro \pstParabolaOrdNode is used to draw a node whose ordinate is the given value y_1 on parabola. Note that \pstParabolaOrdNode will create two nodes A and B at most time.

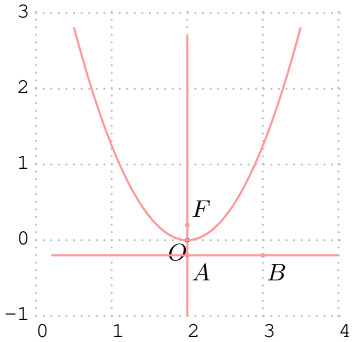
\pstParabolaNode [Options] $(O)\{p\}\{t\}\{A\}$
\pstParabolaAbsNode [Options] $(O)\{p\}\{x_1\}\{A\}$
\pstParabolaOrdNode [Options] $(O)\{p\}\{y_1\}\{A\}\{B\}$




```
\begin{pspicture}[showgrid=true](0,-1)(4,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=-130,PointNameSep=0.2](2,0){O}
\pstParabola[linecolor=red!40](O){\p}{-1.5}{1.5}
\pstParabolaNode[PosAngle=-90](O){\p}{1.5}{A}
\pstParabolaAbsNode[PosAngle=-90,PointName=X_1](O){\p}{1.5}{X1}
\pstParabolaOrdNode[PosAngle=40,PointName={Y_1,Y_2}](O){\p}{1.5}{
    Y1}{Y2}
\end{pspicture}
```

The macro \pstParabolaFocusNode is used to find the focus of the parabola, and the macro \pstParabolaDirectrixLine is used to find the directrix line of the parabola.

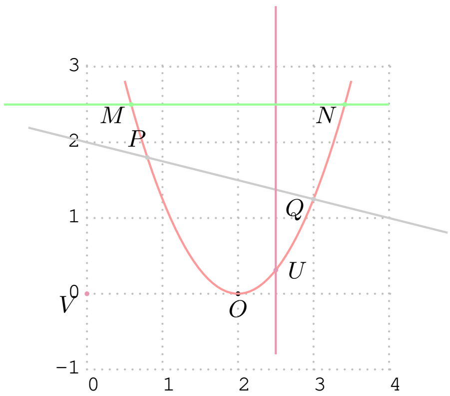
\pstParabolaFocusNode [Options] (O){p}{F}
\pstParabolaDirectrixLine [Options] (O){p}{L_x}{L_y}




```
\begin{pspicture}[showgrid=true](0,-1)(4,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=-130,PointNameSep=0.2](2,0){O}
\pstParabola[linecolor=red!40](O){\p}{-1.5}{1.5}
\pstParabolaFocusNode[linecolor=red!40,PosAngle=50](O){\p}{F}
\pstParabolaDirectrixLine[linecolor=red!40,nodesepA=-1.8,nodesepB
    =-1,PosAngle={-50,-50}](O){\p}{A}{B}
\pstLine[linecolor=red!40,nodesepA=-0.8,nodesepB=-2.5]{A}{F}
\end{pspicture}
```

The macro \pstParabolaLineInter is used to find the intersections C and D of the parabola and the given line AB.

\pstParabolaLineInter [Options] (O){p}{A}{B}{C}{D}




```
\begin{pspicture}[showgrid=true](0,-1)(4,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=-90,PointNameSep=0.2](2,0){O}
\pstParabola[linecolor=red!40](O){\p}{-1.5}{1.5}
\pstLine[linecolor=gray!40,nodesepA=-0.8,nodesepB=-0.8]{0,2}{4,1}
\pstParabolaLineInter[linecolor=gray!40,PosAngle={120,210}](O){\p
    }{0,2}{4,1}{P}{Q}
\pstLine[linecolor=purple!40,nodesepA=-0.8,nodesepB
    =-0.8]{2.5,0}{2.5,3}
\pstParabolaLineInter[linecolor=purple!40,PosAngle={0,210}](O){\p
    }{2.5,0}{2.5,3}{U}{V}
\pstLine[linecolor=green!40,nodesepA=-2.5,nodesepB
    =-1.6]{1.5,2.5}{0.5,2.5}
\pstParabolaLineInter[linecolor=green!40,PosAngle={210,210}](O){\
    p}{1.5,2.5}{0.5,2.5}{M}{N}
\end{pspicture}
```

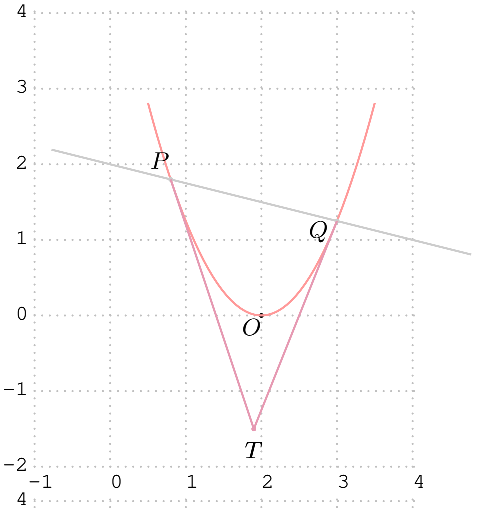
The macro \pstParabolaPolarNode is used to find the polar point T of chord AB on Parabola P.

\pstParabolaPolarNode [Options] (O){p}{A}{B}{T}
\pstParabolaPolarNode [Options] (O){p}(F){A}{B}{T}
\pstParabolaPolarNode [Options] (O){p}(F) [L_x] [L_y] {A}{B}{T}

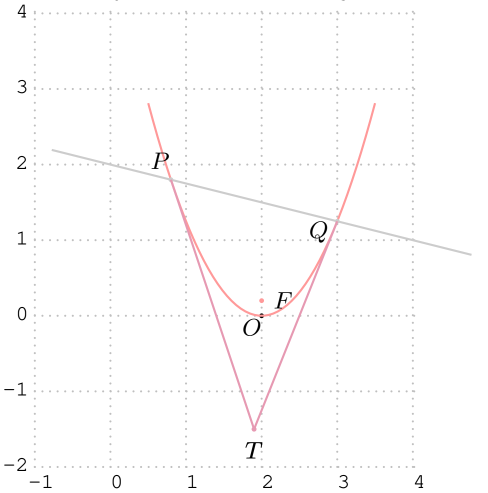
We use the following theorem to find the polar point T of chord AB:

Theorem 3 *Give any chord AB on parabola, drawing two focal chord AFC and BFD, where F is the focus of parabola, then drawing FX which is perpendicular to AFC at point F, and intersect with the directrix line at X; also drawing FY which is perpendicular to BFD at point F, and intersect with the directrix line at Y. Then the intersection T of AX and BY is the polar point of chord AB.*

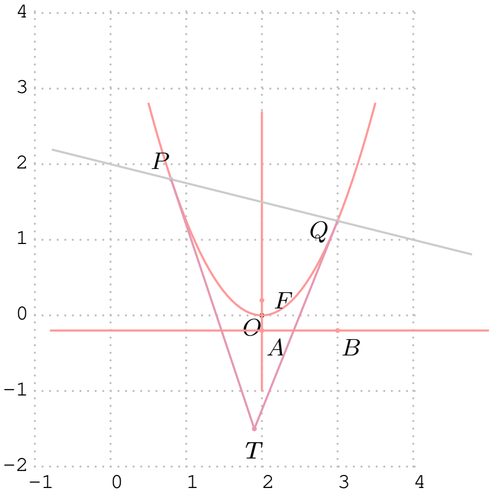
If you don't know the focus F, or the directrix line, we will find them automated, otherwise you can pass them to this macro.




```
\begin{pspicture}[showgrid=true](-1,-2)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=-130,PointNameSep=0.2](2,0){O}
\pstParabola[linecolor=red!40](O){\p}{-1.5}{1.5}
\pstLine[linecolor=gray!40,nodesepA=-0.8,nodesepB=-0.8]{0,2}{4,1}
\pstParabolaLineInter[linecolor=gray!40,PosAngle={120,210}](O){\p
    }{0,2}{4,1}{P}{Q}
% if you don't know focus F or directrix line
\pstParabolaPolarNode[linecolor=purple!40,PosAngle=-90](O){\p}{P
    }{Q}{T}
\end{pspicture}
```




```
\begin{pspicture}[showgrid=true](-1,-2)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=-130,PointNameSep=0.2](2,0){O}
\pstParabola[linecolor=red!40](O){\p}{-1.5}{1.5}
\pstParabolaFocusNode[linecolor=red!40](O){\p}{F}
\pstLine[linecolor=gray!40,nodesepA=-0.8,nodesepB=-0.8]{0,2}{4,1}
\pstParabolaLineInter[linecolor=gray!40,PosAngle={120,210}](O){\p
    }{0,2}{4,1}{P}{Q}
% if you know focus F, but don't know directrix line
\pstParabolaPolarNode[linecolor=purple!40,PosAngle=-90](O){\p}(F)
    {P}{Q}{T}
\end{pspicture}
```



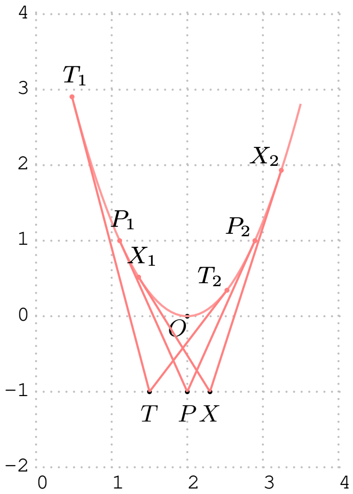

```
\begin{pspicture}[showgrid=true](-1,-2)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=-130,PointNameSep=0.2](2,0){O}
\pstParabola[linecolor=red!40](O){\p}{-1.5}{1.5}
\pstParabolaFocusNode[linecolor=red!40](O){\p}{F}
\pstParabolaDirectrixLine[linecolor=red!40,nodesepA=-2.8,nodesepB
    =-2,PosAngle={-50,-50}](O){\p}{A}{B}
\pstLineAB[linecolor=red!40,nodesepA=-0.8,nodesepB=-2.5]{A}{F}
\pstLine[linecolor=gray!40,nodesepA=-0.8,nodesepB=-0.8]{0,2}{4,1}
\pstParabolaLineInter[linecolor=gray!40,PosAngle={120,210}](O){\p
    }{0,2}{4,1}{P}{Q}
% if you know focus F and also directrix line
\pstParabolaPolarNode[linecolor=purple!40,PosAngle=-90](O){\p}(F)
    [A][B]{P}{Q}{T}
\end{pspicture}
```

The macro `\pstParabolaTangentNode` is used to find the two nodes A and B on the Parabola through the point T.

`\pstParabolaTangentNode [Options]` $(O)\{p\}\{T\}\{A\}\{B\}$

We use the following theorem to find the tangent node A and B of outside point T:

Theorem 4 *Give point T outside of the parabola, we draw any other two chords TPQ and TRS, PS and QR intersect at I, PR and QS intersect at X, XI and Parabola intersect at A and B, then TA is the tangent line through A and TB is the tangent line through B.*




```
\begin{pspicture}[showgrid=true](0,-2)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=-130,PointNameSep=0.2](2,0){O}
\pstParabola[linecolor=red!40](O){\p}{-1.5}{1.5}
\pstGeonode[PosAngle=-90](1.5,-1){T}
\pstParabolaTangentNode[linecolor=red!50,PosAngle={80,140},
    PointName={T_1,T_2}](O){\p}{T}{T1}{T2}
\pstGeonode[PosAngle=-90](2,-1){P}
\pstParabolaTangentNode[linecolor=red!50,PosAngle={80,140},
    PointName={P_1,P_2}](O){\p}{P}{P1}{P2}
\pstGeonode[PosAngle=-90](2.3,-1){X}
\pstParabolaTangentNode[linecolor=red!50,PosAngle={80,140},
    PointName={X_1,X_2}](O){\p}{X}{X1}{X2}
\end{pspicture}
```

3.4. Standard Conjugate Parabola

The Conjugate Parabola P with coordinate translation is defined by vertex $O(x_0, y_0)$, the half of the focus chord axis $abs(p)$. Note that the sign of p indicates the direction of the parabola. The equation can be written as:

$$(y - y_0)^2 = 2p(x - x_0) \tag{8}$$

and the parametric function can be written as:

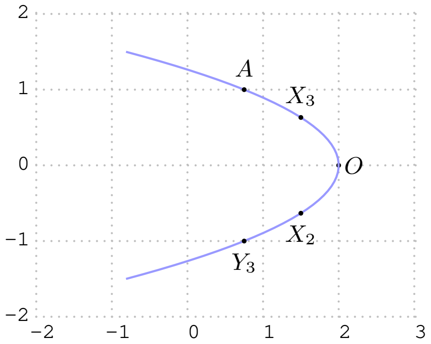
$$\begin{cases} x = \dfrac{t^2}{2p} + x_0 \\ y = t + y_0 \end{cases} \tag{9}$$

The macro `\pstIParabola` is used to draw a Standard Conjugate Parabola from y_1 to y_2 with Vertex O, the half of the focus chord axis $abs(p)$.

`\pstIParabola [Options] (`O`){`p`}{`y_1`}{`y_2`}`

The macro `\pstIParabolaNode` is used to draw a node whose parameter is the given value t on parabola, please refer to equation (9). The macro `\pstIParabolaAbsNode` is used to draw a node whose abscissa is the given value x_1 on parabola. The macro `\pstIParabolaOrdNode` is used to draw a node whose ordinate is the given value y_1 on parabola. Note that `\pstIParabolaAbsNode` will create two nodes A and B at most time.

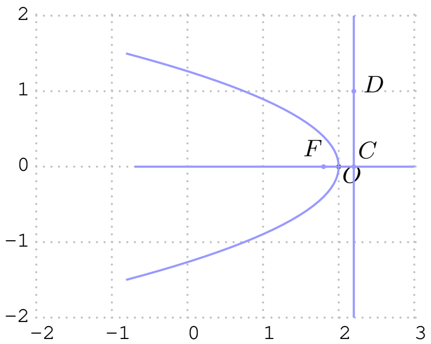
`\pstIParabolaNode [Options] (`O`){`p`}{`t`}{`A`}`
`\pstIParabolaAbsNode [Options] (`O`){`p`}{`x_1`}{`A`}{`B`}`
`\pstIParabolaOrdNode [Options] (`O`){`p`}{`y_1`}{`A`}`




```
\begin{pspicture}[showgrid=true](-2,-2)(3,2)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=0,PointNameSep=0.2](2,0){O}
\pstIParabola[linecolor=blue!40](O){-\p}{-1.5}{1.5}
\pstIParabolaNode[PosAngle=90](O){-\p}{1}{A}
\pstIParabolaAbsNode[PosAngle=90,PointName={X_2,X_3},PosAngle
    ={-90,90}](O){-\p}{1.5}{X2}{X3}
\pstIParabolaOrdNode[PosAngle=-90,PointName=Y_3](O){-\p}{-1}{Y3}
\end{pspicture}
```

The macro \pstIParabolaFocusNode is used to find the focus of the parabola, and the macro \pstIParabolaDirectrixLine is used to find the directrix line of the parabola.

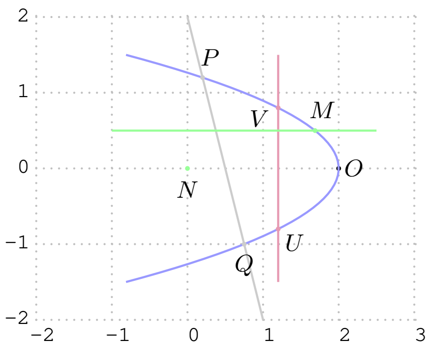
\pstIParabolaFocusNode [Options] $(O)\{p\}\{F\}$
\pstIParabolaDirectrixLine [Options] $(O)\{p\}\{L_x\}\{L_y\}$




```
\begin{pspicture}[showgrid=true](-2,-2)(3,2)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=-30,PointNameSep=0.2](2,0){O}
\pstIParabola[linecolor=blue!40](O){-\p}{-1.5}{1.5}
\pstIParabolaFocusNode[linecolor=blue!40,PosAngle=120](O){-\p}{F}
\pstIParabolaDirectrixLine[linecolor=blue!40,nodesepA=-2,nodesepB
    =-1,PosAngle={50,20}](O){-\p}{C}{D}
\pstLine[linecolor=blue!40,nodesepA=-0.8,nodesepB=-2.5]{C}{F}
\end{pspicture}
```

The macro \pstIParabolaLineInter is used to find the intersections C and D of the parabola and the given line AB.

\pstIParabolaLineInter [Options] $(O)\{p\}\{A\}\{B\}\{C\}\{D\}$

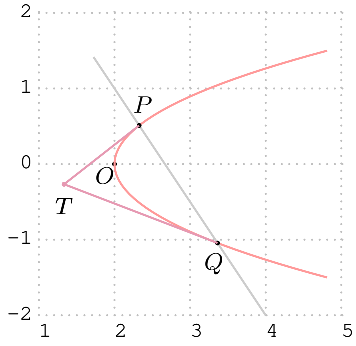



```
\begin{pspicture}[showgrid=true](-2,-2)(3,2)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=0,PointNameSep=0.2](2,0){O}
\pstIParabola[linecolor=blue!40](O){-\p}{-1.5}{1.5}
\pstLine[linecolor=gray!40]{0,2}{1,-2}
\pstIParabolaLineInter[linecolor=gray!40,PosAngle={70,-90}](O){-\
    p}{1,-2}{0,2}{P}{Q}
\pstLine[linecolor=purple!40]{1.2,-1.5}{1.2,1.5}
\pstIParabolaLineInter[linecolor=purple!40,PosAngle={-40,210}](O)
    {-\p}{1.2,-1.5}{1.2,1.5}{U}{V}
\pstLine[linecolor=green!40]{-1,0.5}{2.5,0.5}
\pstIParabolaLineInter[linecolor=green!40,PosAngle={70,-90}](O)
    {-\p}{-1,0.5}{2.5,0.5}{M}{N}
\end{pspicture}
```

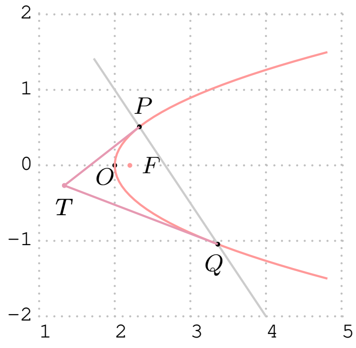
The macro \pstIParabolaPolarNode is used to find the polar point T of chord AB on Parabola P.

\pstIParabolaPolarNode [Options] $(O)\{p\}\{A\}\{B\}\{T\}$
\pstIParabolaPolarNode [Options] $(O)\{p\}(F)\{A\}\{B\}\{T\}$
\pstIParabolaPolarNode [Options] $(O)\{p\}(F)$ $[L_x]$ $[L_y]$ $\{A\}\{B\}\{T\}$

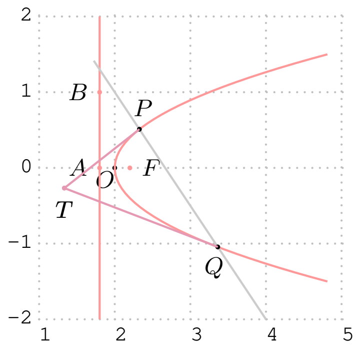
We also use the theorem 3 to find the polar point T of chord AB. If you don't know the focus F, or the directrix line, we will find them automated, otherwise you can pass them to this macro.




```
\begin{pspicture}[showgrid=true](1,-2)(5,2)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=-130,PointNameSep=0.2](2,0){O}
\pstIParabola[linecolor=red!40](O){\p}{-1.5}{1.5}
\pstLine[linecolor=gray!40,nodesepA=-0.5]{2,1}{4,-2}
\pstIParabolaLineInter[PosAngle={80,-100}](O){\p}{2,1}{4,-2}{P}{Q
    }
% if you don't know focus F or directrix line
\pstIParabolaPolarNode[linecolor=purple!40,PosAngle=-90](O){\p}{P
    }{Q}{T}
\end{pspicture}
```




```
\begin{pspicture}[showgrid=true](1,-2)(5,2)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=-130,PointNameSep=0.2](2,0){O}
\pstIParabola[linecolor=red!40](O){\p}{-1.5}{1.5}
\pstIParabolaFocusNode[linecolor=red!40](O){\p}{F}
\pstLine[linecolor=gray!40,nodesepA=-0.5]{2,1}{4,-2}
\pstIParabolaLineInter[PosAngle={80,-100}](O){\p}{2,1}{4,-2}{P}{Q
    }
% if you know focus F, but don't known directrix line
\pstIParabolaPolarNode[linecolor=purple!40,PosAngle=-90](O){\p}(F
    ){P}{Q}{T}
\end{pspicture}
```

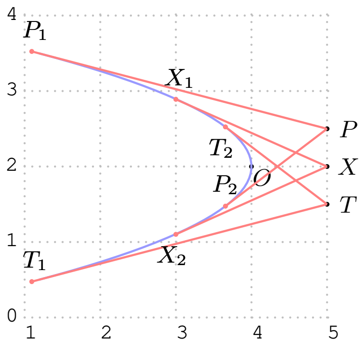



```
\begin{pspicture}[showgrid=true](1,-2)(5,2)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=-130,PointNameSep=0.2](2,0){O}
\pstIParabola[linecolor=red!40](O){\p}{-1.5}{1.5}
\pstIParabolaFocusNode[linecolor=red!40](O){\p}{F}
\pstIParabolaDirectrixLine[linecolor=red!40,nodesepA=-2,nodesepB
    =-1,PosAngle={180,180}](O){\p}{A}{B}
\pstLine[linecolor=gray!40,nodesepA=-0.5]{2,1}{4,-2}
\pstIParabolaLineInter[PosAngle={80,-100}](O){\p}{2,1}{4,-2}{P}{Q
    }
% if you know focus F and also directrix line
\pstIParabolaPolarNode[linecolor=purple!40,PosAngle=-90](O){\p}(F
    )[A][B]{P}{Q}{T}
\end{pspicture}
```

The macro `\pstIParabolaTangentNode` is used to find the two nodes A and B on the Parabola through the point T.

`\pstIParabolaTangentNode [Options]` $(O)\{p\}\{T\}\{A\}\{B\}$

We also use the theorem 4 to find the tangent node A and B of outside point T!




```
\begin{pspicture}[showgrid=true](1,0)(5,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=-45,PointNameSep=0.2](4,2){O}
\pstIParabola[linecolor=blue!40](O){-\p}{-1.5}{1.5}
\pstGeonode[PosAngle=0](5,1.5){T}
\pstIParabolaTangentNode[linecolor=red!50,PosAngle={80,-100},
    PointName={T_1,T_2}](O){-\p}{T}{T1}{T2}
\pstGeonode[PosAngle=0](5,2.5){P}
\pstIParabolaTangentNode[linecolor=red!50,PosAngle={80,90},
    PointName={P_1,P_2}](O){-\p}{P}{P1}{P2}
\pstGeonode[PosAngle=0](5,2){X}
\pstIParabolaTangentNode[linecolor=red!50,PosAngle={80,-100},
    PointName={X_1,X_2}](O){-\p}{X}{X1}{X2}
\end{pspicture}
```

3.5. General Parabola

The General Parabola P with coordinate translation and rotation is defined by vertex $O(x_0, y_0)$, the half of the focus chord axis $abs(p)$, the sign of p indicates the direction of the parabola, and the rotation angle θ of the symmetrical axis. The symmetrical axis is perpendicular to x-axis when $\theta = 0°$, and perpendicular to y-axis when $\theta = 90°$.

The equation can be got from the parametric function of the parabola equation (7), using the rotation transform formula (3), then we have

$$\begin{cases} x' = (t + x_0)\cos\theta - (\dfrac{t^2}{2p} + y_0)\sin\theta = x'_0 + t\cos\theta - t^2\dfrac{\sin\theta}{2p} \\ y' = (t + x_0)\sin\theta + (\dfrac{t^2}{2p} + y_0)\cos\theta = y'_0 + t\sin\theta + t^2\dfrac{\cos\theta}{2p} \end{cases} \tag{10}$$

where the x'_0 and y'_0 are the coordinate of the given vertex O after rotation. So we get the parametric function of the General Parabola with coordinate translation and rotation as following:

$$\begin{cases} x = x_0 + t\cos\theta - t^2\dfrac{\sin\theta}{2p} \\ y = y_0 + t\sin\theta + t^2\dfrac{\cos\theta}{2p} \end{cases} \tag{11}$$

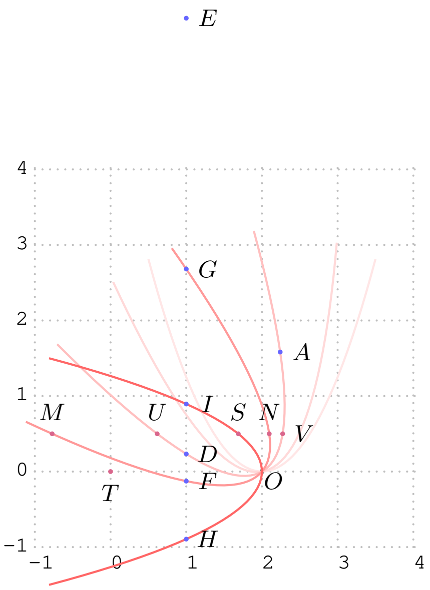
The macro \pstGeneralParabola is used to draw a General Parabola from x_1 to x_2 with Vertex O, the half of the focus chord axis $abs(p)$.

```
\pstGeneralParabola [Options] (O){p} [θ] {x1}{x2}
```

The macro \pstGeneralParabolaNode is used to draw a node whose parameter is the given value t on parabola, please refer to equation (11). The macro \pstGeneralParabolaAbsNode is used to draw a node whose abscissa is the given value x_1 on parabola. The macro \pstGeneralParabolaOrdNode is used to draw a node whose ordinate is the given value y_1 on parabola.

Note that \pstGeneralParabolaAbsNode and \pstGeneralParabolaOrdNode both create two nodes A and B at most time.

```
\pstGeneralParabolaNode [Options] (O){p} [θ] {t}{A}
\pstGeneralParabolaAbsNode [Options] (O){p} [θ] {x1}{A}{B}
\pstGeneralParabolaOrdNode [Options] (O){p} [θ] {y1}{A}{B}
```

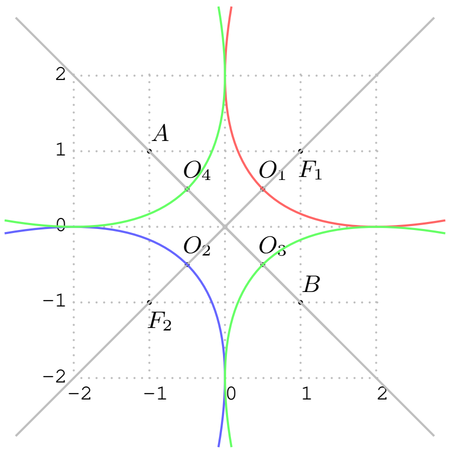



```
\begin{pspicture}[showgrid=true](-1,-1)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=-40,PointNameSep=0.2](2,0){O}
\pstGeneralParabola[linecolor=red!10](O){\p}[0]{-1.5}{1.5}
\pstGeneralParabola[linecolor=red!15](O){\p}[10]{-1.5}{1.5}
\pstGeneralParabola[linecolor=red!25](O){\p}[30]{-1.5}{1.5}
\pstGeneralParabola[linecolor=red!40](O){\p}[50]{-1.5}{1.5}
\pstGeneralParabola[linecolor=red!60](O){\p}[90]{-1.5}{1.5}
\pstGeneralParabolaNode[PosAngle=0,linecolor=blue!60](O){\p
    }[30]{1.0}{A}
\pstGeneralParabolaAbsNode[PosAngle={0,0},linecolor=blue!60](O){\
    p}[30]{1.0}{D}{E}
\pstGeneralParabolaAbsNode[PosAngle={0,0},linecolor=blue!60](O){\
    p}[50]{1.0}{F}{G}
\pstGeneralParabolaAbsNode[PosAngle={0,0},linecolor=blue!60](O){\
    p}[90]{1.0}{H}{I}
\pstGeneralParabolaOrdNode[PosAngle={90,0},linecolor=purple!60](O
    ){\p}[30]{0.5}{U}{V}
\pstGeneralParabolaOrdNode[PosAngle={90,90},linecolor=purple!60](
    O){\p}[50]{0.5}{M}{N}
\pstGeneralParabolaOrdNode[PosAngle={90,-90},linecolor=purple
    !60](O){\p}[90]{0.5}{S}{T}
\end{pspicture}
```

The Macro \pstGeneralParabolaFl is used to define a General Parabola with Focus F, and the directrix line l. It just calculate the vertex O, half focal chord p, and the rotation angle θ of the symmetrical axis, then you can pass them into macro \pstGeneralParabola to draw this parabola.

\pstGeneralParabolaFl [Options] $\{F\}\{A\}\{B\}\{O\}\{p\}\{\theta\}$

The output parameter O is a node name to store the vertex point, its label and symbol can be controlled by the options for PSTricks node, such as PosAngle. The output parameter p is a PostScript key to store the value of half focal chord. The output parameter θ is also a PostScript key to store the rotation angle of symmetrical axis, when you pass it to \pstGeneralParabola, PostScript will lookup the value of this key in current dictionary.



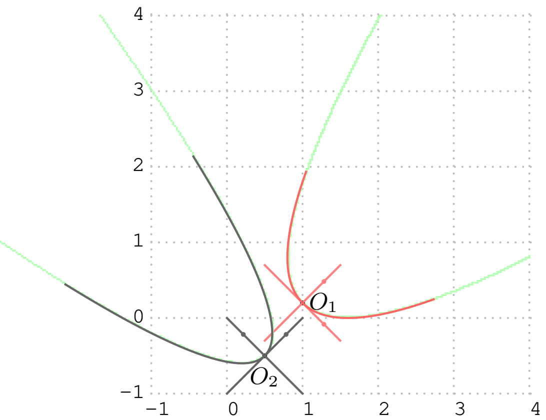

```
\begin{pspicture}[showgrid=true](-2,-2)(2,2)
\psset{unit=1.0cm}\psset{dotscale=0.5}\footnotesize
\psset{CodeFig=true,CodeFigColor=gray!50}\psset{PointSymbol=*}
\pstGeonode[PosAngle=-60](1,1){F_1}
\pstGeonode[PosAngle=-60](-1,-1){F_2}
\pstGeonode[PosAngle=60](1,-1){B}
\pstGeonode[PosAngle=60](-1,1){A}
\pstGeneralParabolaFl[PosAngle=60]{F_1}{A}{B}{O_1}{
    semifocalchordp1}{SymAxisRotAngle1}
\pstGeneralParabola[linecolor=red!60](O_1){semifocalchordp1}[
    SymAxisRotAngle1]{-2}{2}
\pstGeneralParabolaFl[PosAngle=60]{F_2}{A}{B}{O_2}{
    semifocalchordp2}{SymAxisRotAngle2}
\pstGeneralParabola[linecolor=blue!60](O_2){semifocalchordp2}[
    SymAxisRotAngle2]{-2}{2}
\pstGeneralParabolaFl[PosAngle=60]{B}{F_1}{F2}{O_3}{
    semifocalchordp3}{SymAxisRotAngle3}
\pstGeneralParabola[linecolor=green!60](O_3){semifocalchordp3}[
    SymAxisRotAngle3]{-2}{2}
\pstGeneralParabolaFl[PosAngle=60]{A}{F_1}{F2}{O_4}{
    semifocalchordp4}{SymAxisRotAngle4}
\pstGeneralParabola[linecolor=green!60](O_4){semifocalchordp4}[
    SymAxisRotAngle4]{-2}{2}
\end{pspicture}
```

The Macro `\pstGeneralParabolaCoef` is used to define a General Parabola by the quadratic curve equation $ax^2 + bxy + cy^2 + dx + ey + f = 0$, it just calculate the vertex O, half focal chord p and the rotation angle θ of the symmetrical axis, then you can pass them into macro `\pstGeneralParabola` to draw this parabola. The package `pst-func` provides macro `\psplotImp` to draw an implicit defined functions too, but it can't tell you the geometrical elements like as center or radii, and it will take more time to calculate the function value point by point.

`\pstGeneralParabolaCoef [Options]` {a,b,c,d,e,f}{O}{p}{θ}

The output parameter `O`, `p` and $\theta$are same with `\pstGeneralParabolaFl`. They are set to zero if the coeffients are invalid to construct a parabola. If you pass the zero p into macro `\pstGeneralParabola`, it will abort with the exception of dividing by zero.

In the following example, we use `\psplotImp` to draw the same parabolas, just to check the results given by macros `\pstGeneralParabolaCoef` are correct.



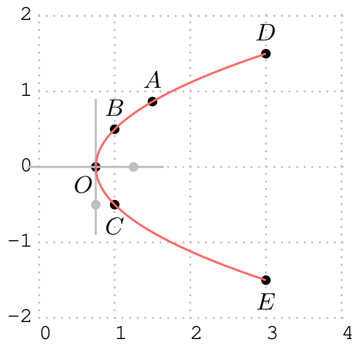

```
\begin{pspicture}[showgrid=true](-1,-1)(4,4)
\psset{unit=0.40cm}\footnotesize\psset{dotscale=0.5}
\psset{CodeFig=true}\psset{PointSymbol=*}
%x^2-2xy+y^2-8x+16=0
\psplotImp[linecolor=green!30](-10,-10)(10,10){ 1 x dup mul mul
    -2 x mul y mul add 1 y dup mul mul add -8 x mul add 0 y mul
    add 16 add }
\pstGeneralParabolaCoef[PosAngle=0,CodeFigColor=red
    !50]{1,-2,1,-8,0,16}{O_1}{P1}{SymAxisRotAngle1}
\pstGeneralParabola[linecolor=red!60](O_1){P1}[SymAxisRotAngle
    1]{-3}{3}
%x^2+2xy+y^2+2x-2y-5=0
\psplotImp[linecolor=green!30](-10,-10)(10,10){ 1 x dup mul mul 2
     x mul y mul add 1 y dup mul mul add 2 x mul add -2 y mul
    add -5 add }
\pstGeneralParabolaCoef[PosAngle=-90,CodeFigColor=black
    !60]{1,2,1,2,-2,-5}{O_2}{P2}{SymAxisRotAngle2}
\pstGeneralParabola[linecolor=black!60](O_2){P2}[SymAxisRotAngle
    2]{-3}{3}
\end{pspicture}
```

The Macro \pstGeneralParabolaABCDE is used to define a General Parabola by the given five points A, B, C, D, E, it just calculate the vertex O, half focal chord p and the rotation angle θ of the symmetrical axis, then you can pass them into macro \pstGeneralParabola to draw this parabola.

\pstGeneralParabolaABCDE [Options] $\{A\}\{B\}\{C\}\{D\}\{E\}\{O\}\{p\}\{\theta\}$

The output parameter O, p and θ are same with \pstGeneralParabolaFl. They are set to zero if the points are invalid to construct a parabola. If you pass the zero p into macro \pstGeneralParabola, it will abort with the exception of dividing by zero.

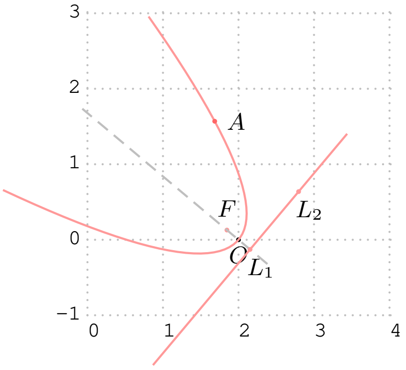
Note the algorithm may fit a hyperbola quadratic curve from the given five points, in order to get the right parabola curve, you must input the point coordinates very precisely. In the following example, if you input point A as $(3, 1.732)$, it will fail as no such parabola can fit these five points.




```
\begin{pspicture}[showgrid=true](0,-2)(4,2)
\psset{unit=0.5cm}\footnotesize\psset{PointSymbol=*}
\psset{CodeFig=true,CodeFigColor=gray!50}
% five points from y^2-2x+3=0
\pstGeonode[PosAngle=90](3,1.73205){A}
\pstGeonode[PosAngle=90](2,1){B}
\pstGeonode[PosAngle=-90](2,-1){C}
\pstGeonode[PosAngle=90](6,3){D}
\pstGeonode[PosAngle=-90](6,-3){E}
\pstGeneralParabolaABCDE[PosAngle=235]{A}{B}{C}{D}{E}{O}{P}{
    SymAxisRotAngle}
\pstGeneralParabola[linecolor=red!60](O){P}[SymAxisRotAngle
    ]{-3}{3}
\end{pspicture}
```

The macro \pstGeneralParabolaFocusNode is used to find the focus of the parabola, and the macro \pstGeneralParabolaDirectrixLine is used to find the directrix line of the parabola.

\pstGeneralParabolaFocusNode [Options] $(O)\{p\}$ [θ] $\{F\}$
\pstGeneralParabolaDirectrixLine [Options] $(O)\{p\}$ [θ] $\{L_x\}\{L_y\}$

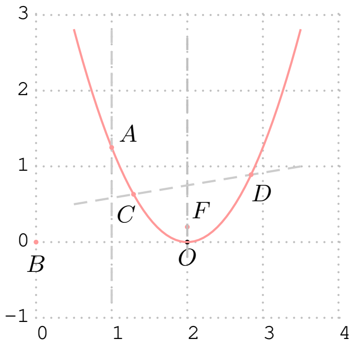



```
\begin{pspicture}[showgrid=true](0,-1)(4,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=-90,PointNameSep=0.2](2,0){O}
\pstGeneralParabola[linecolor=red!40](O){\p}[50]{-1.5}{1.5}
\pstGeneralParabolaFocusNode[linecolor=red!40,PosAngle=90](O){\p
    }[50]{F}
\pstLineAB[linestyle=dashed,linecolor=black!25,nodesepA=-0.5,
    nodesepB=-2.5]{O}{F}
\pstGeneralParabolaDirectrixLine[linecolor=red!40,nodesepA=-2,
    nodesepB=-1,PosAngle={-60,-60},PointName={L_1,L_2}](O){\p
    }[50]{L1}{L2}
\pstGeneralParabolaNode[linecolor=red!60](O){\p}[50]{1.0}{A}
\end{pspicture}
```

The macro `\pstGeneralParabolaLineInter` is used to find the intersections C and D of the parabola and the given line AB.

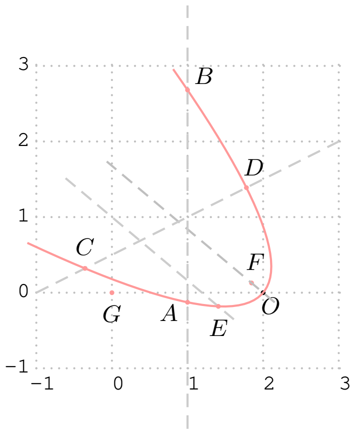
`\pstGeneralParabolaLineInter [Options]` $(O)\{p\}$ $[\theta]$ $\{A\}\{B\}\{C\}\{D\}$

When General Parabola becomes a Standard Parabola, the intersections with any kind of lines:



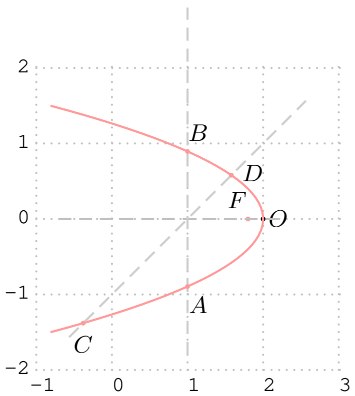

```
\begin{pspicture}[showgrid=true](0,-1)(4,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=-90,PointNameSep=0.2](2,0){O}
\pstGeneralParabola[linecolor=red!40](O){\p}[0]{-1.5}{1.5}
\pstGeneralParabolaFocusNode[linecolor=red!40,PosAngle=50](O){\p
    }[0]{F}
\pstLineAB[linestyle=dashed,linecolor=black!25,nodesepA=-0.2,
    nodesepB=-2.5]{O}{F}
\pstLine[linestyle=dashed,linecolor=gray!40,nodesep
    =-0.8]{1,0}{1,2}
\pstGeneralParabolaLineInter[linecolor=red!40,PosAngle={40,-90}](
    O){\p}[0]{1,0}{1,2}{A}{B}
\pstLine[linestyle=dashed,linecolor=gray!40,nodesep
    =0]{0.5,0.5}{3.5,1}
\pstGeneralParabolaLineInter[linecolor=red!40,PosAngle
    ={-110,-60}](O){\p}[0]{0.5,0.5}{3.5,1}{C}{D}
\end{pspicture}
```

Here is the intersections of a real General Parabola with any kind of lines:




```
\begin{pspicture}[showgrid=true](-1,-1)(3,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=-60,PointNameSep=0.2](2,0){O}
\pstGeneralParabola[linecolor=red!40](O){\p}[50]{-1.5}{1.5}
\pstGeneralParabolaFocusNode[linecolor=red!40,PosAngle=80](O){\p
    }[50]{F}
\pstLineAB[linestyle=dashed,linecolor=black!25,nodesepA=-0.2,
    nodesepB=-2.5]{O}{F}
\pstLine[linestyle=dashed,linecolor=gray!40,nodesep
    =-0.8]{1,-1}{1,3}
\pstGeneralParabolaLineInter[linecolor=red!40,PosAngle
    ={-150,40}](O){\p}[50]{1,-1}{1,3}{A}{B}
\pstLine[linestyle=dashed,linecolor=gray!40,nodesep
    =0.0]{-1,0}{3,2}
\pstGeneralParabolaLineInter[linecolor=red!40,PosAngle={90,70}](O
    ){\p}[50]{-1,0}{3,2}{C}{D}
% a line with gradient k=-\cos50/\sin50 parallel to OF
\pstLineAS[linestyle=dashed,linecolor=gray!40,nodesep=-0.8,
    PointName=none,PointSymbol=none](0,1){50 cos 50 sin div neg
    }{X}
\pstGeneralParabolaLineInter[linecolor=red!40,PosAngle
    ={-90,-90}](O){\p}[50]{0,1}{X}{E}{G}
\end{pspicture}
```

When General Parabola becomes a Standard Conjugate Parabola, the intersections with any kind of lines:

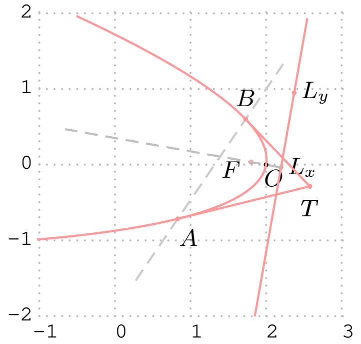



```
\begin{pspicture}[showgrid=true](-1,-2)(3,2)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=0,PointNameSep=0.2](2,0){O}
\pstGeneralParabola[linecolor=red!40](O){\p}[90]{-1.5}{1.5}
\pstGeneralParabolaFocusNode[linecolor=red!40,PosAngle=120](O){\p
    }[90]{F}
\pstLineAB[linestyle=dashed,linecolor=black!25,nodesepA=-0.2,
    nodesepB=-2.5]{O}{F}
\pstLine[linestyle=dashed,linecolor=gray!40,nodesep
    =-0.8]{1,-1}{1,2}
\pstGeneralParabolaLineInter[linecolor=red!40,PosAngle={-60,60}](
    O){\p}[90]{1,-1}{1,2}{A}{B}
\pstLine[linestyle=dashed,linecolor=gray!40,nodesep
    =-0.8]{0,-1}{2,1}
\pstGeneralParabolaLineInter[linecolor=red!40,PosAngle={-90,5}](O
    ){\p}[90]{0,-1}{2,1}{C}{D}
\end{pspicture}
```

The macro `\pstGeneralParabolaPolarNode` is used to find the polar point T of chord AB on Parabola P.

`\pstGeneralParabolaPolarNode` [Options] $(O)\{p\}$ [θ] $\{A\}\{B\}\{T\}$
`\pstGeneralParabolaPolarNode` [Options] $(O)\{p\}$ [θ] $(F)\{A\}\{B\}\{T\}$
`\pstGeneralParabolaPolarNode` [Options] $(O)\{p\}$ [θ] (F) [L_x] [L_y] $\{A\}\{B\}\{T\}$

We also use the theorem 3 to find the polar point T of chord AB. If you don't know the focus F, or the directrix line, we will find them automated, otherwise you can pass them to this macro.

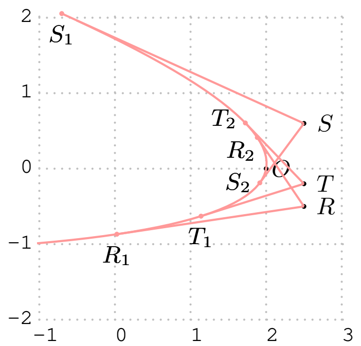



```
\begin{pspicture}[showgrid=true](-1,-2)(3,2)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=-60,PointNameSep=0.2](2,0){O}
\pstGeneralParabola[linecolor=red!40](O){\p}[80]{-1.5}{1.5}
\pstGeneralParabolaFocusNode[linecolor=red!40,PosAngle=200](O){\p
    }[80]{F}
\pstGeneralParabolaDirectrixLine[linecolor=red!40,nodesepA=-2,
    nodesepB=-1,PosAngle={0,0},PointName={L_x,L_y}](O){\p}[80]{
    Lx}{Ly}
\pstLine[linestyle=dashed,linecolor=black!25,nodesepA=-0.2,
    nodesepB=-2.5]{O}{F}
\pstLine[linestyle=dashed,linecolor=gray!40,nodesep
    =-0.4]{0.5,-1.2}{2,1}
\pstGeneralParabolaLineInter[linecolor=red!40,PosAngle={-60,90}](
    O){\p}[80]{0.5,-1.2}{2,1}{A}{B}
%\pstGeneralParabolaPolarNode[linecolor=red!40,PosAngle=-90](O){\
    p}[80]{A}{B}{T}
%\pstGeneralParabolaPolarNode[linecolor=red!40,PosAngle=-90](O){\
    p}[80](F){A}{B}{T}
\pstGeneralParabolaPolarNode[linecolor=red!40,PosAngle=-90](O){\p
    }[80](F)[Lx][Ly]{A}{B}{T}
\end{pspicture}
```

The macro `\pstGeneralParabolaTangentNode` is used to find the two nodes A and B on the Parabola through the point T.

`\pstGeneralParabolaTangentNode [Options]` $(O)\{p\}$ $[\theta]$ $\{T\}\{A\}\{B\}$

We also use the theorem 4 to find the tangent node A and B of outside point T.




```
\begin{pspicture}[showgrid=true](-1,-2)(3,2)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=0,PointNameSep=0.2](2,0){O}
\pstGeneralParabola[linecolor=red!40](O){\p}[80]{-1.5}{1.5}
\pstGeonode[PosAngle=0](2.5,-0.5){R}(2.5,-0.2){T}(2.5,0.6){S}
\pstGeneralParabolaTangentNode[linecolor=red!40,PosAngle
    ={-90,220},PointName={R_1,R_2}](O){\p}[80]{R}{R1}{R2}
\pstGeneralParabolaTangentNode[linecolor=red!40,PosAngle
    ={-90,170},PointName={T_1,T_2}](O){\p}[80]{T}{T1}{T2}
\pstGeneralParabolaTangentNode[linecolor=red!40,PosAngle
    ={-90,180},PointName={S_1,S_2}](O){\p}[80]{S}{S1}{S2}
\end{pspicture}
```

3.6. General Conjugate Parabola

The General Conjugate Parabola P with coordinate translation and rotation is defined by vertex O, the half of the focus chord axis $abs(p)$, the sign of p indicates the direction of the parabola, and the rotation angle θ of the symmetrical axis.

The equation can be got from the parametric function of the conjugate parabola (9), using the rotation transform formula (3), then we have

$$\begin{cases} x' = (\dfrac{t^2}{2p} + x_0)\cos\theta - (t + y_0)\sin\theta = x'_0 - t\sin\theta + t^2\dfrac{\cos\theta}{2p} \\ y' = (\dfrac{t^2}{2p} + x_0)\sin\theta + (t + y_0)\cos\theta = y'_0 + t\cos\theta + t^2\dfrac{\sin\theta}{2p} \end{cases} \tag{12}$$

where the x_0' and y_0' are the coordinate of the given vertex O after rotation. So we get the parametric function of the General Conjugate Parabola with coordinate translation and rotation as following:

$$\begin{cases} x = x_0 - t\sin\theta + t^2\dfrac{\cos\theta}{2p} \\ y = y_0 + t\cos\theta + t^2\dfrac{\sin\theta}{2p} \end{cases} \tag{13}$$

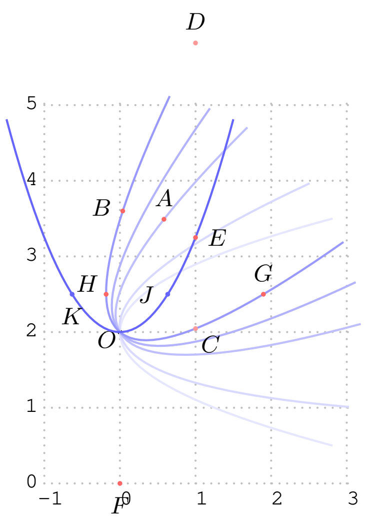
The macro `\pstGeneralIParabola` is used to draw a Standard Conjugate Parabola from y_1 to y_2 with Vertex O, the half of the focus chord axis $abs(p)$.

`\pstGeneralIParabola [Options] (`O`){`p`} [`θ`] {`y_1`}{`y_2`}`

The macro `\pstGeneralIParabolaNode` is used to draw a node whose parameter is the given value t on parabola, please refer to equation (13). The macro `\pstGeneralIParabolaAbsNode` is used to draw a node whose abscissa is the given value x_1 on parabola. The macro `\pstGeneralIParabolaOrdNode` is used to draw a node whose ordinate is the given value y_1 on parabola.

Note that `\pstGeneralIParabolaAbsNode` and `\pstGeneralIParabolaOrdNode` will create two nodes A and B at most time.

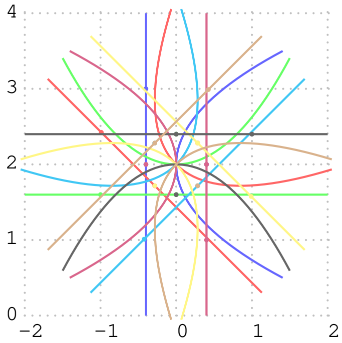
`\pstGeneralIParabolaNode [Options] (`O`){`p`} [`θ`] {`t`}{`A`}`
`\pstGeneralIParabolaAbsNode [Options] (`O`){`p`} [`θ`] {`x_1`}{`A`}{`B`}`
`\pstGeneralIParabolaOrdNode [Options] (`O`){`p`} [`θ`] {`y_1`}{`A`}{`B`}`




```
\begin{pspicture}[showgrid=true](-1,0)(3,5)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=210,PointNameSep=0.2](0,2){O}
\pstGeneralIParabola[linecolor=blue!10](O){\p}[0]{-1.5}{1.5}
\pstGeneralIParabola[linecolor=blue!15](O){\p}[10]{-1.5}{1.5}
\pstGeneralIParabola[linecolor=blue!25](O){\p}[30]{-1.5}{1.5}
\pstGeneralIParabola[linecolor=blue!30](O){\p}[40]{-1.5}{1.5}
\pstGeneralIParabola[linecolor=blue!40](O){\p}[50]{-1.5}{1.5}
\pstGeneralIParabola[linecolor=blue!60](O){\p}[90]{-1.5}{1.5}
\pstGeneralIParabolaNode[linecolor=red!60,PosAngle=90](O){\p
    }[30]{1.0}{A}
\pstGeneralIParabolaNode[linecolor=red!60,PosAngle=170](O){\p
    }[50]{1.0}{B}
\pstGeneralIParabolaAbsNode[linecolor=red!40,PosAngle={-45,90}](O
    ){\p}[50]{1.0}{C}{D}
\pstGeneralIParabolaAbsNode[linecolor=red!60,PosAngle={0,-90}](O)
    {\p}[90]{1.0}{E}{F}
\pstGeneralIParabolaOrdNode[linecolor=red!60,PosAngle={90,150}](O
    ){\p}[50]{2.5}{G}{H}
\pstGeneralIParabolaOrdNode[linecolor=blue!60,PosAngle
    ={180,-90}](O){\p}[90]{2.5}{J}{K}
\end{pspicture}
```

The macro `\pstGeneralIParabolaFocusNode` is used to find the focus of the parabola, and the macro `\pstGeneralIParabolaDirectrixLine` is used to find the directrix line of the parabola.

`\pstGeneralIParabolaFocusNode [Options] (`O`){`p`} [`θ`] {`F`}`
`\pstGeneralIParabolaDirectrixLine [Options] (`O`){`p`} [`θ`] {`L_x`}{`L_y`}`

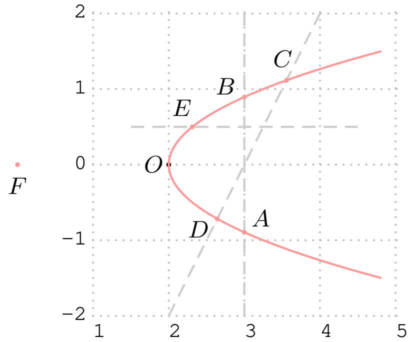



```
\begin{pspicture}[showgrid=true](-2,0)(2,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\psset{PointName=none,nodesepA=-2,nodesepB=-1}
\pstGeonode(0,2){O}\def\p{0.8}
\psset{linecolor=blue!60}
\pstGeneralIParabola(O){\p}[0]{-1.5}{1.5}
\pstGeneralIParabolaFocusNode(O){\p}[0]{A}
\pstGeneralIParabolaDirectrixLine(O){\p}[0]{A1}{A2}
\psset{linecolor=red!60}
\pstGeneralIParabola(O){\p}[45]{-1.5}{1.5}
\pstGeneralIParabolaFocusNode(O){\p}[45]{B}
\pstGeneralIParabolaDirectrixLine(O){\p}[45]{B1}{B2}
\psset{linecolor=green!60}
\pstGeneralIParabola(O){\p}[90]{-1.5}{1.5}
\pstGeneralIParabolaFocusNode(O){\p}[90]{C}
\pstGeneralIParabolaDirectrixLine(O){\p}[90]{C1}{C2}
\psset{linecolor=cyan!60}
\pstGeneralIParabola(O){\p}[135]{-1.5}{1.5}
\pstGeneralIParabolaFocusNode(O){\p}[135]{D}
\pstGeneralIParabolaDirectrixLine(O){\p}[135]{D1}{D2}
\psset{linecolor=purple!60}
\pstGeneralIParabola(O){\p}[180]{-1.5}{1.5}
\pstGeneralIParabolaFocusNode(O){\p}[180]{E}
\pstGeneralIParabolaDirectrixLine(O){\p}[180]{E1}{E2}
\psset{linecolor=yellow!60}
\pstGeneralIParabola(O){\p}[225]{-1.5}{1.5}
\pstGeneralIParabolaFocusNode(O){\p}[225]{F}
\pstGeneralIParabolaDirectrixLine(O){\p}[225]{F1}{F2}
\psset{linecolor=black!60}
\pstGeneralIParabola(O){\p}[270]{-1.5}{1.5}
\pstGeneralIParabolaFocusNode(O){\p}[270]{G}
\pstGeneralIParabolaDirectrixLine(O){\p}[270]{G1}{G2}
\psset{linecolor=brown!60}
\pstGeneralIParabola(O){\p}[315]{-1.5}{1.5}
\pstGeneralIParabolaFocusNode(O){\p}[315]{H}
\pstGeneralIParabolaDirectrixLine(O){\p}[315]{H1}{H2}
\end{pspicture}
```

The macro `\pstGeneralIParabolaLineInter` is used to find the intersections C and D of the parabola and the given line AB.

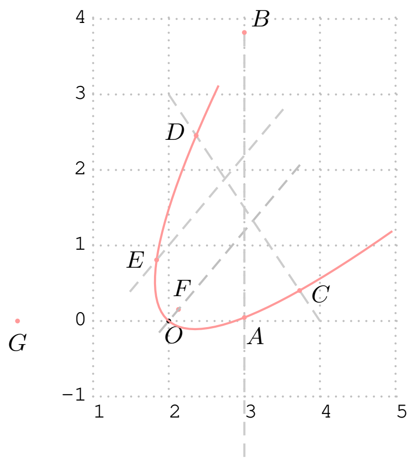
`\pstGeneralIParabolaLineInter [Options] (`O`){`p`} [`θ`] {`A`}{`B`}{`C`}{`D`}`

When $\theta = 0$, the intersections with any kind of lines:



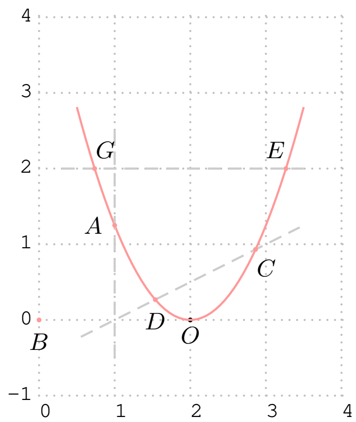

```
\begin{pspicture}[showgrid=true](1,-2)(5,2)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=180,PointNameSep=0.2](2,0){O}
\pstGeneralIParabola[linecolor=red!40](O){\p}[0]{-1.5}{1.5}
\pstLine[linestyle=dashed,linecolor=gray!40]{3,-2}{3,2}
\pstGeneralIParabolaLineInter[linecolor=red!40,PosAngle
    ={40,150}](O){\p}[0]{3,-2}{3,2}{A}{B}
\pstLine[linestyle=dashed,linecolor=gray!40]{2,-2}{4,2}
\pstGeneralIParabolaLineInter[linecolor=red!40,PosAngle
    ={100,210}](O){\p}[0]{2,-2}{4,2}{C}{D}
\pstLine[linestyle=dashed,linecolor=gray!40]{1.5,0.5}{4.5,0.5}
\pstGeneralIParabolaLineInter[linecolor=red!40,PosAngle
    ={120,-90}](O){\p}[0]{1.5,0.5}{4.5,0.5}{E}{F}
\end{pspicture}
```

When $\theta = 50$, the intersections with any kind of lines:




```
\begin{pspicture}[showgrid=true](1,-1)(5,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=-70,PointNameSep=0.2](2,0){O}
\pstGeneralIParabola[linecolor=red!40](O){\p}[50]{-1.5}{1.5}
\pstGeneralIParabolaFocusNode[linecolor=red!40,PosAngle=80](O){\p
    }[50]{F}
\pstLineAB[linestyle=dashed,linecolor=black!25,nodesepA=-0.2,
    nodesepB=-2.5]{O}{F}
\pstLine[linestyle=dashed,linecolor=gray!40,nodesep
    =-0.8]{3,-1}{3,3}
\pstGeneralIParabolaLineInter[linecolor=red!40,PosAngle
    ={-60,40}](O){\p}[50]{3,-1}{3,3}{A}{B}
\pstLine[linestyle=dashed,linecolor=gray!40,nodesep
    =0.0]{2,3}{4,0}
\pstGeneralIParabolaLineInter[linecolor=red!40,PosAngle
    ={-10,170}](O){\p}[50]{2,3}{4,0}{C}{D}
% a line with gradient k=\tan50 parallel to OF
\pstLineAS[linestyle=dashed,linecolor=gray!40,nodesep=-0.8,
    PointName=none,PointSymbol=none](2,1){50 tan}{X}
\pstGeneralIParabolaLineInter[linecolor=red!40,PosAngle
    ={180,-90}](O){\p}[50]{2,1}{X}{E}{G}
\end{pspicture}
```

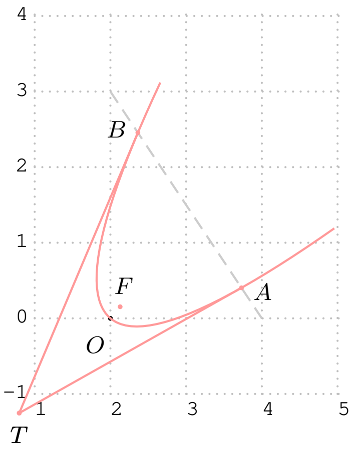
When $\theta = 90$, the intersections with any kind of lines:




```
\begin{pspicture}[showgrid=true](0,-1)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=-90,PointNameSep=0.2](2,0){O}
\pstGeneralIParabola[linecolor=red!40](O){\p}[90]{-1.5}{1.5}
\pstLine[linestyle=dashed,linecolor=gray!40,nodesep
    =-0.5]{1,0}{1,2}
\pstGeneralIParabolaLineInter[linecolor=red!40,PosAngle
    ={180,-90}](O){\p}[90]{1,0}{1,2}{A}{B}
\pstLine[linestyle=dashed,linecolor=gray!40,nodesep
    =-0.5]{1,0}{3,1}
\pstGeneralIParabolaLineInter[linecolor=red!40,PosAngle
    ={-60,-90}](O){\p}[90]{1,0}{3,1}{C}{D}
\pstLine[linestyle=dashed,linecolor=gray!40,nodesep
    =-0.5]{0.8,2}{3,2}
\pstGeneralIParabolaLineInter[linecolor=red!40,PosAngle
    ={120,60}](O){\p}[90]{0.8,2}{3,2}{E}{G}
\end{pspicture}
```

The macro `\pstGeneralIParabolaPolarNode` is used to find the polar point T of chord AB on Parabola P.

\pstGeneralIParabolaPolarNode [Options] $(O)\{p\}$ $[\theta]$ $\{A\}\{B\}\{T\}$
\pstGeneralIParabolaPolarNode [Options] $(O)\{p\}$ $[\theta]$ $(F)\{A\}\{B\}\{T\}$
\pstGeneralIParabolaPolarNode [Options] $(O)\{p\}$ $[\theta]$ (F) $[L_x]$ $[L_y]$ $\{A\}\{B\}\{T\}$

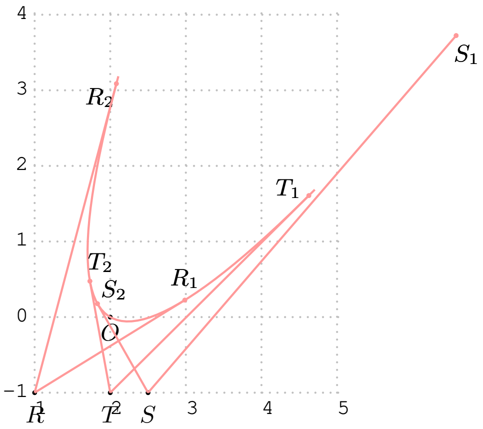



```
\begin{pspicture}[showgrid=true](1,-1)(5,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=240,PointNameSep=0.4](2,0){O}
\pstGeneralIParabola[linecolor=red!40](O){\p}[50]{-1.5}{1.5}
\pstGeneralIParabolaFocusNode[linecolor=red!40,PosAngle=80](O){\p
    }[50]{F}
\pstLine[linestyle=dashed,linecolor=gray!40,nodesep
    =0.0]{2,3}{4,0}
\pstGeneralIParabolaLineInter[linecolor=red!40,PosAngle
    ={-10,170}](O){\p}[50]{2,3}{4,0}{A}{B}
\pstGeneralIParabolaPolarNode[linecolor=red!40,PosAngle=-90](O){\
    p}[50](F){A}{B}{T}
\end{pspicture}
```

We also use the theorem 3 to find the polar point T of chord AB. If you don't know the focus F, or the directrix line, we will find them automated, otherwise you can pass them to this macro.

The macro `\pstGeneralIParabolaTangentNode` is used to find the two nodes A and B on the Parabola through the point T.

`\pstGeneralIParabolaTangentNode` `[Options]` $(O)\{p\}$ `[`θ`]` $\{T\}\{A\}\{B\}$




```
\begin{pspicture}[showgrid=true](1,-1)(5,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\p{0.4}
\pstGeonode[PosAngle=-90,PointNameSep=0.2](2,0){O}
\pstGeneralIParabola[linecolor=red!40](O){\p}[60]{-1.5}{1.5}
\pstGeonode[PosAngle=-90](1,-1){R}(2,-1){T}(2.5,-1){S}
\pstGeneralIParabolaTangentNode[linecolor=red!40,PosAngle
    ={90,220},PointName={R_1,R_2}](O){\p}[60]{R}{R1}{R2}
\pstGeneralIParabolaTangentNode[linecolor=red!40,PosAngle
    ={160,60},PointName={T_1,T_2}](O){\p}[60]{T}{T1}{T2}
\pstGeneralIParabolaTangentNode[linecolor=red!40,PosAngle
    ={-60,40},PointName={S_1,S_2}](O){\p}[60]{S}{S1}{S2}
\end{pspicture}
```

3.7. Standard Hyperbola

The Standard Hyperbola H with coordinate translation is defined by center O, the half of the real axis a, the half of the imaginary axis b. The equation can be written as:

$$\frac{(x-x_0)^2}{a^2}-\frac{(y-y_0)^2}{b^2}=1 \tag{14}$$

and the parametric function can be written as:

$$\begin{cases} x = a\sec\alpha + x_0 \\ y = b\tan\alpha + y_0 \end{cases} \tag{15}$$

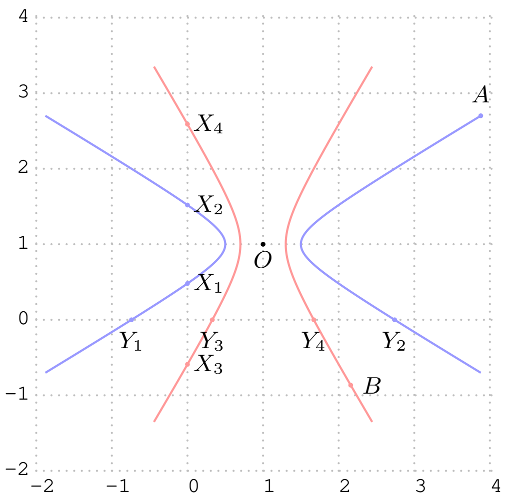
The macro `\pstHyperbola` is used to draw a Standard Hyperbola with Center O, the half of the real axis a, the half of the imaginary axis b. The parameter `angleX` is used to truncate the width of the figure, it should be setup from 0 to 90.

`\pstHyperbola` `[Options]` $(O)(a,\,b)$ `[angleX]`

The macro `\pstHyperbolaNode` is used to draw a node whose parameter is the given value t on Hyperbola, please refer to equation (15). The macro `\pstHyperbolaAbsNode` is used to draw the nodes whose abscissa are the given value x_1 on Hyperbola. The macro `\pstHyperbolaOrdNode` is used to draw the nodes whose ordinate are the given value y_1 on Hyperbola.

Note that `\pstHyperbolaAbsNode` and `\pstHyperbolaOrdNode` will create two nodes A and B at most time.

`\pstHyperbolaNode` `[Options]` $(O)(a,\ b)\{t\}\{A\}$
`\pstHyperbolaAbsNode` `[Options]` $(O)(a,\ b)\{x_1\}\{A\}\{B\}$
`\pstHyperbolaOrdNode` `[Options]` $(O)(a,\ b)\{y_1\}\{A\}\{B\}$



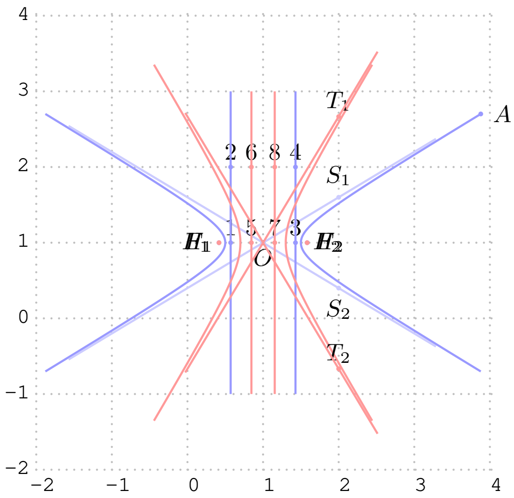

```
\begin{pspicture}[showgrid=true](-2,-2)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\a{0.5}\def\b{0.3}
\pstGeonode[PosAngle=-90,PointNameSep=0.2](1,1){O}
\pstHyperbola[linecolor=blue!40](O)(\a,\b)[80]
\pstHyperbolaNode[linecolor=blue!40,PosAngle=90](O)(\a,\b){80}{A}
\pstHyperbolaAbsNode[linecolor=blue!40,PointName={X_1,X_2},
    PosAngle=0](O)(\a,\b){0}{X1}{X2}
\pstHyperbolaOrdNode[linecolor=blue!40,PointName={Y_1,Y_2},
    PosAngle=-90](O)(\a,\b){0}{Y1}{Y2}
\pstHyperbola[linecolor=red!40](O)(\b,\a)[78]
\pstHyperbolaNode[linecolor=red!40](O)(\b,\a){-75}{B}
\pstHyperbolaAbsNode[linecolor=red!40,PointName={X_3,X_4},
    PosAngle=0](O)(\b,\a){0}{X3}{X4}
\pstHyperbolaOrdNode[linecolor=red!40,PointName={Y_3,Y_4},
    PosAngle=-90](O)(\b,\a){0}{Y3}{Y4}
\end{pspicture}
```

The macro `\pstHyperbolaFocusNode` is used to find the focus nodes of the Hyperbola, and the macro `\pstHyperbolaDirectrixLine` is used to find the directrix lines of the Hyperbola.

`\pstHyperbolaFocusNode` `[Options]` $(O)(a,\ b)\{F_1\}\{F_2\}$
`\pstHyperbolaDirectrixLine` `[Options]` $(O)(a,\ b)\{L_x\}\{L_y\}\{R_x\}\{R_y\}$

Note that you can use `\pstLineAS` to draw the asymptote line of the hyperbola by passing the slope gradient $k = \pm\dfrac{b}{a}$; or you can use the macro `\pstHyperbolaAsymptoteLine` to get them, this macro only create one node on each asymptote line, as the other one is the center of the hyperbola.

`\pstHyperbolaAsymptoteLine` `[Options]` $(O)(a,\ b)\{L_1\}\{L_2\}$

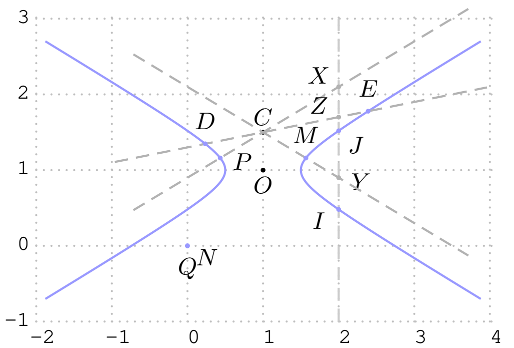


```
\begin{pspicture}[showgrid=true](-2,-2)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\a{0.5}\def\b{0.3}
\pstGeonode[PosAngle=-90,PointNameSep=0.2](1,1){O}
\pstHyperbola[linecolor=blue!40](O)(\a,\b)[80]
\pstHyperbolaNode[linecolor=blue!40](O)(\a,\b){80}{A}
\pstLineAS[PointName=S_1,PosAngle=90,nodesepA=-3,nodesepB=-1.5,
    linecolor=blue!20]{O}{\b\space \a\space div}{S1}
\pstLineAS[PointName=S_2,PosAngle=-90,nodesepA=-3,nodesepB=-1.5,
    linecolor=blue!20]{O}{\b\space \a\space div neg}{S2}
\pstHyperbolaFocusNode[linecolor=blue!40,PointName={F_1,F_2},
    PosAngle={180,0}](O)(\a,\b){F1}{F2}
\pstHyperbolaDirectrixLine[linecolor=blue!40,nodesepA=-2,nodesepB
    =-1,PointName={1,2,3,4},PosAngle=90,PointNameSep=0.2](O)(\a
    ,\b){Lx}{Ly}{Rx}{Ry}
\pstHyperbola[linecolor=red!40](O)(\b,\a)[78]
\pstHyperbolaFocusNode[linecolor=red!40,PointName={H_1,H_2},
    PosAngle={180,0}](O)(\b,\a){H1}{H2}
\pstHyperbolaDirectrixLine[linecolor=red!40,nodesepA=-2,nodesepB
    =-1,PointName={5,6,7,8},PosAngle=90,PointNameSep=0.2](O)(\b
    ,\a){Mx}{My}{Nx}{Ny}
\pstHyperbolaAsymptoteLine[linecolor=red!40,nodesepA=-2,nodesepB
    =-1,PointName={T_1,T_2},PosAngle=90,PointNameSep=0.2](O)(\b
    ,\a){T1}{T2}
\end{pspicture}
```

The macro \pstHyperbolaLineInter is used to find the intersections C and D of the hyperbola and the given line AB.

\pstHyperbolaLineInter [Options] $(O)(a, b)\{A\}\{B\}\{C\}\{D\}$

In the following example, the Line CX and CY are parallel to the asymptote of the hyperbola.



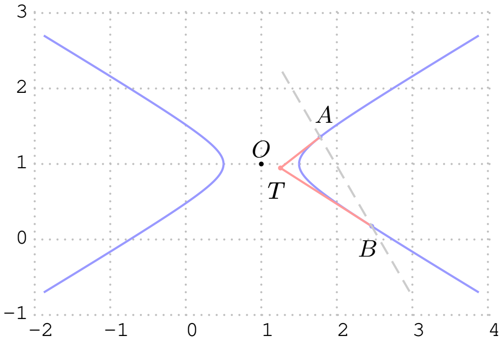
```
\begin{pspicture}[showgrid=true](-2,-1)(4,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\a{0.5}\def\b{0.3}\psset{PointNameSep=0.3}
\pstGeonode[PosAngle={-90,90},PointNameSep=0.2](1,1){O}(1,1.5){C}
\pstHyperbola[linecolor=blue!40](O)(\a,\b)[80]
\pstLine[linestyle=dashed,linecolor=gray!40]{2,-1}{2,3}
\pstHyperbolaLineInter[linecolor=blue!40,PosAngle={210,-40}](O)(\
    a,\b){2,-1}{2,3}{I}{J}
\pstLineAS[linestyle=dashed,linecolor=gray!60,nodesep=-2,PosAngle
    =150]{1,1.5}{\b\space \a\space div}{X}
\pstLineAS[linestyle=dashed,linecolor=gray!60,nodesep=-2,PosAngle
    =-10]{1,1.5}{\b\space \a\space div neg}{Y}
\pstLineAS[linestyle=dashed,linecolor=gray!60,nodesep=-2,PosAngle
    =150]{1,1.5}{0.2}{Z}
\pstHyperbolaLineInter[linecolor=blue!40,PosAngle={-10,-90}](O)(\
    a,\b){1,1.5}{X}{P}{Q}
\pstHyperbolaLineInter[linecolor=blue!40,PosAngle={90,-30}](O)(\a
    ,\b){1,1.5}{Y}{M}{N}
\pstHyperbolaLineInter[linecolor=blue!40,PosAngle={90,90}](O)(\a
    ,\b){1,1.5}{Z}{D}{E}
\end{pspicture}
```

The macro \pstHyperbolaPolarNode is used to find the polar point T of chord AB on the hyperbola.

\pstHyperbolaPolarNode [Options] $(O)(a, b)\{A\}\{B\}\{T\}$

We use the following theorem to find the polar point T of chord AB:

Theorem 5 *Let P, Q are vertex points of the hyperbola, for any chord AB of hyperbola, suppose PA and BQ intersect at E, PB and AQ intersect at F, then the middle point T of EF is the polar point of chord AB.*



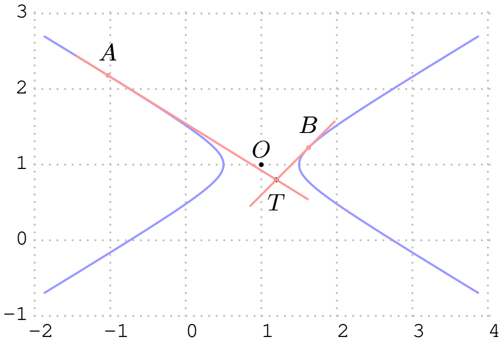

```
\begin{pspicture}[showgrid=true](-2,-1)(4,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\a{0.5}\def\b{0.3}\psset{PointNameSep=0.3}
\pstGeonode[PosAngle=90,PointNameSep=0.2](1,1){O}
\pstHyperbola[linecolor=blue!40](O)(\a,\b)[80]
\pstHyperbolaNode[linecolor=blue!40,PosAngle=80](O)(\a,\b){50}{A}
\pstHyperbolaNode[linecolor=blue!40,PosAngle=-100](O)(\a,\b)
    {-70}{B}
\pstHyperbolaPolarNode[linecolor=red!40,PosAngle=-100](O)(\a,\b){
    A}{B}{T}
\pstLine[linestyle=dashed,linecolor=gray!40,nodesep=-1]{A}{B}
\end{pspicture}
```

The macro `\pstHyperbolaTangentNode` is used to find the tangent point A and B of point T outside of the hyperbola.

`\pstHyperbolaTangentNode [Options]` $(O)(a,b)\{T\}\{A\}\{B\}$

We use the following theorem to find the tangent points A and B of T:

Theorem 6 *Let T is a point out of the hyperbola, for any two chords TPQ and TRS of the hyperbola, suppose PR and QS intersect at X, RQ and PS intersect at Y, then the intersection points A and B of XY and the hyperbola are the tangent points from T.*




```
\begin{pspicture}[showgrid=true](-2,-1)(4,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\a{0.5}\def\b{0.3}\psset{PointNameSep=0.3}
\pstGeonode[PosAngle=90,PointNameSep=0.2](1,1){O}
\pstHyperbola[linecolor=blue!40](O)(\a,\b)[80]
\pstGeonode[PosAngle=-90](1.2,0.8){T}
\pstHyperbolaTangentNode[linecolor=red!40,PosAngle={90,90},
    nodesep=-0.5](O)(\a,\b){T}{A}{B}
\end{pspicture}
```

3.8. Standard Conjugate Hyperbola

The Standard Conjugate Hyperbola H with coordinate translation is defined by center O, the half of the real axis a, the half of the imaginary axis b. The equation can be written as:

$$\frac{(y-y_0)^2}{a^2}-\frac{(x-x_0)^2}{b^2}=1 \tag{16}$$

and the parametric function can be written as:

$$\begin{cases} x = b\tan\alpha + x_0 \\ y = a\sec\alpha + y_0 \end{cases} \tag{17}$$

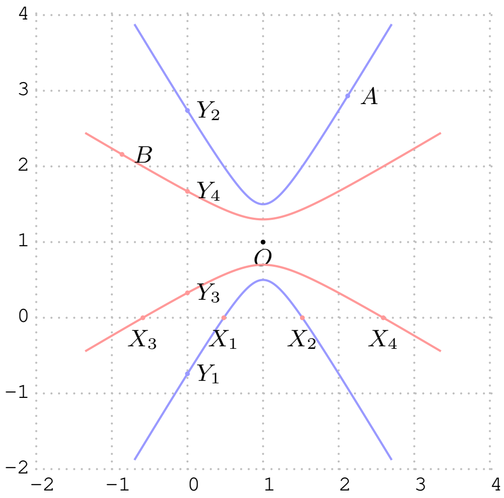
The macro `\pstIHyperbola` is used to draw a Standard Conjugate Hyperbola with Center O, the half of the real axis a, the half of the imaginary axis b. The parameter `angleY` is used to truncate the height of the figure, it should be setup from 0 to 90.

\pstIHyperbola [Options] (O)(a, b) [angleY]

The macro `\pstIHyperbolaNode` is used to draw a node whose parameter is the given value t on Conjugate Hyperbola, please refer to equation (17). The macro `\pstIHyperbolaAbsNode` is used to draw the nodes whose abscissa are the given value x_1 on Conjugate Hyperbola. The macro `\pstIHyperbolaOrdNode` is used to draw the nodes whose ordinate are the given value y_1 on Conjugate Hyperbola.

Note that `\pstIHyperbolaAbsNode` and `\pstIHyperbolaOrdNode` will create two nodes A and B at most time.

\pstIHyperbolaNode [Options] (O)(a, b){t}{A}
\pstIHyperbolaAbsNode [Options] (O)(a, b){x_1}{A}{B}
\pstIHyperbolaOrdNode [Options] (O)(a, b){y_1}{A}{B}



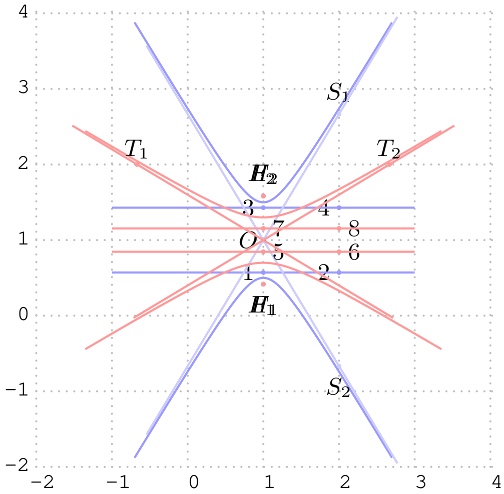

```
\begin{pspicture}[showgrid=true](-2,-2)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\a{0.5}\def\b{0.3}
\pstGeonode[PosAngle=-90,PointNameSep=0.2](1,1){O}
\pstIHyperbola[linecolor=blue!40](O)(\a,\b)[80]
\pstIHyperbolaNode[linecolor=blue!40](O)(\a,\b){75}{A}
\pstIHyperbolaAbsNode[linecolor=blue!40,PointName={Y_1,Y_2},
    PosAngle=0](O)(\a,\b){0}{Y1}{Y2}
\pstIHyperbolaOrdNode[linecolor=red!40,PointName={X_1,X_2},
    PosAngle=-90](O)(\a,\b){0}{X1}{X2}
\pstIHyperbola[linecolor=red!40](O)(\b,\a)[78]
\pstIHyperbolaNode[linecolor=red!40](O)(\b,\a){-75}{B}
\pstIHyperbolaAbsNode[linecolor=red!40,PointName={Y_3,Y_4},
    PosAngle=0](O)(\b,\a){0}{Y3}{Y4}
\pstIHyperbolaOrdNode[linecolor=red!40,PointName={X_3,X_4},
    PosAngle=-90](O)(\b,\a){0}{X3}{X4}
\end{pspicture}
```

The macro `\pstIHyperbolaFocusNode` is used to find the focus nodes of the Conjugate Hyperbola, and the macro `\pstIHyperbolaDirectrixLine` is used to find the directrix lines of the Conjugate Hyperbola.

\pstIHyperbolaFocusNode [Options] (O)(a, b){F_1}{F_2}
\pstIHyperbolaDirectrixLine [Options] (O)(a, b){L_x}{L_y}{R_x}{R_y}

Note that you can use `\pstLineAS` to draw the asymptote line of the hyperbola by passing the slope gradient $k = \pm\frac{a}{b}$; or you can use the macro `\pstIHyperbolaAsymptoteLine` to get them, this macro only create one node on each asymptote line, as the other one is the center of the hyperbola.

\pstHyperbolaAsymptoteLine [Options] (O)(a, b){L_1}{L_2}

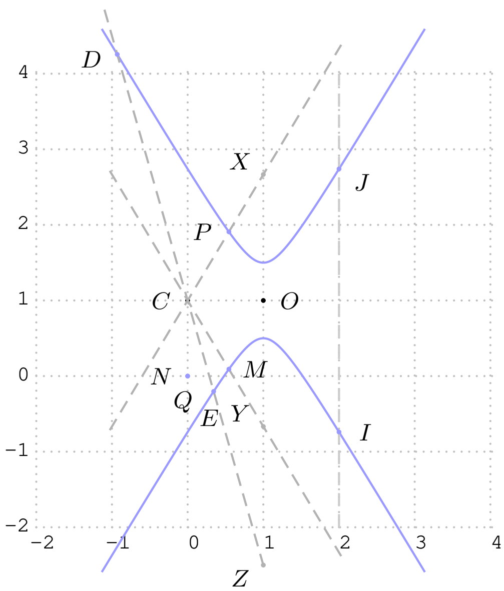


```
\begin{pspicture}[showgrid=true](-2,-2)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\a{0.5}\def\b{0.3}
\pstGeonode[PosAngle=180,PointNameSep=0.2](1,1){O}
\pstIHyperbola[linecolor=blue!40](O)(\a,\b)[80]
\pstIHyperbolaFocusNode[linecolor=blue!40,PointName={F_1,F_2},
    PosAngle={-90,90}](O)(\a,\b){F1}{F2}
\pstIHyperbolaDirectrixLine[linecolor=blue!40,nodesepA=-2,
    nodesepB=-1,PointName={1,2,3,4},PosAngle=180,PointNameSep
    =0.2](O)(\a,\b){Lx}{Ly}{Rx}{Ry}
\pstLineAS[PointName=S_1,PosAngle=90,nodesepA=-3,nodesepB=-1.5,
    linecolor=blue!20]{O}{\a\space \b\space div}{S1}
\pstLineAS[PointName=S_2,PosAngle=-90,nodesepA=-3,nodesepB=-1.5,
    linecolor=blue!20]{O}{\a\space \b\space div neg}{S2}
\pstIHyperbola[linecolor=red!40](O)(\b,\a)[78]
\pstIHyperbolaFocusNode[linecolor=red!40,PointName={H_1,H_2},
    PosAngle={-90,90}](O)(\b,\a){H1}{H2}
\pstIHyperbolaDirectrixLine[linecolor=red!40,nodesepA=-2,nodesepB
    =-1,PointName={5,6,7,8},PosAngle=0,PointNameSep=0.2](O)(\b,\
    a){Mx}{My}{Nx}{Ny}
\pstIHyperbolaAsymptoteLine[linecolor=red!40,nodesepA=-2,nodesepB
    =-1,PointName={T_1,T_2},PosAngle=90,PointNameSep=0.2](O)(\b
    ,\a){T1}{T2}
\end{pspicture}
```

The macro `\pstIHyperbolaLineInter` is used to find the intersections C and D of the hyperbola and the given line AB.

`\pstIHyperbolaLineInter [Options] (`O`)(`a, b`){`A`}{`B`}{`C`}{`D`}`

In the following example, the Line CX and CY are parallel to the asymptote of the hyperbola.

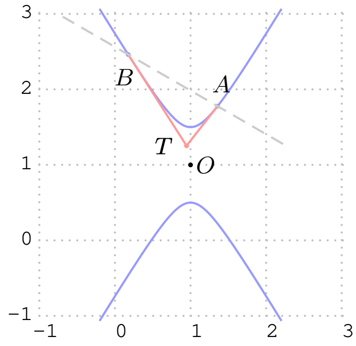


```
\begin{pspicture}[showgrid=true](-2,-2)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\a{0.5}\def\b{0.3}\psset{PointNameSep=0.35}
\pstGeonode[PosAngle={0,180}](1,1){O}(0,1){C}
\pstIHyperbola[linecolor=blue!40](O)(\a,\b)[82]
\pstLine[linestyle=dashed,linecolor=gray!40]{2,-2}{2,4}
\pstIHyperbolaLineInter[linecolor=blue!40,PosAngle={0,-30}](O)(\a
    ,\b){2,-2}{2,4}{I}{J}
\pstLineAS[linestyle=dashed,linecolor=gray!60,nodesep=-2,PosAngle
    =150]{0,1}{\a\space \b\space div}{X}
\pstLineAS[linestyle=dashed,linecolor=gray!60,nodesep=-2,PosAngle
    =150]{0,1}{\a\space \b\space div neg}{Y}
\pstLineAS[linestyle=dashed,linecolor=gray!60,nodesepA=-4,
    PosAngle=210]{0,1}{-3.5}{Z}
\pstIHyperbolaLineInter[linecolor=blue!40,PosAngle={180,-100}](O)
    (\a,\b){0,1}{X}{P}{Q}
\pstIHyperbolaLineInter[linecolor=blue!40,PosAngle={0,180}](O)(\a
    ,\b){0,1}{Y}{M}{N}
\pstIHyperbolaLineInter[linecolor=blue!40,PosAngle={190,-100}](O)
    (\a,\b){0,1}{Z}{D}{E}
\end{pspicture}
```

The macro `\pstIHyperbolaPolarNode` is used to find the polar point T of chord AB on the hyperbola.

`\pstIHyperbolaPolarNode [Options] (`O`)(`a, b`){`A`}{`B`}{`T`}`

We also use the theorem 5 to find the polar point T of chord AB:

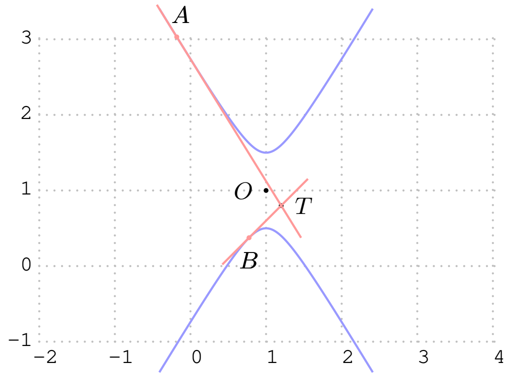



```
\begin{pspicture}[showgrid=true](-1,-1)(3,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\a{0.5}\def\b{0.3}\psset{PointNameSep=0.3}
\pstGeonode[PosAngle=0,PointNameSep=0.2](1,1){O}
\pstIHyperbola[linecolor=blue!40](O)(\a,\b)[76]
\pstIHyperbolaNode[linecolor=blue!40,PosAngle=80](O)(\a,\b){50}{A
    }
\pstIHyperbolaNode[linecolor=blue!40,PosAngle=-100](O)(\a,\b)
    {-70}{B}
\pstIHyperbolaPolarNode[linecolor=red!40,PosAngle=180](O)(\a,\b){
    A}{B}{T}
\pstLine[linestyle=dashed,linecolor=gray!40,nodesep=-1]{A}{B}
\end{pspicture}
```

The macro `\pstIHyperbolaTangentNode` is used to find the tangent point A and B of point T outside of the hyperbola.

`\pstIHyperbolaTangentNode [Options]` $(O)(a,\, b)\{T\}\{A\}\{B\}$

We also use the theorem 6 to find the tangent points A and B of T.




```
\begin{pspicture}[showgrid=true](-2,-1)(4,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\a{0.5}\def\b{0.3}\psset{PointNameSep=0.3}
\pstGeonode[PosAngle=180](1,1){O}
\pstIHyperbola[linecolor=blue!40](O)(\a,\b)[78]
\pstGeonode[PosAngle=0](1.2,0.8){T}
\pstIHyperbolaTangentNode[linecolor=red!40,PosAngle={80,-90},
    nodesep=-0.5](O)(\a,\b){T}{A}{B}
\end{pspicture}
```

3.9. General Hyperbola

The General Hyperbola H with coordinate translation and rotation is defined by center O, the half of the real axis a, the half of the imaginary axis b, and the rotation angle θ of the principal axis. The equation can be got from the parametric function of the Standard Hyperbola equation (15), using the rotation transform formula (3), then we have

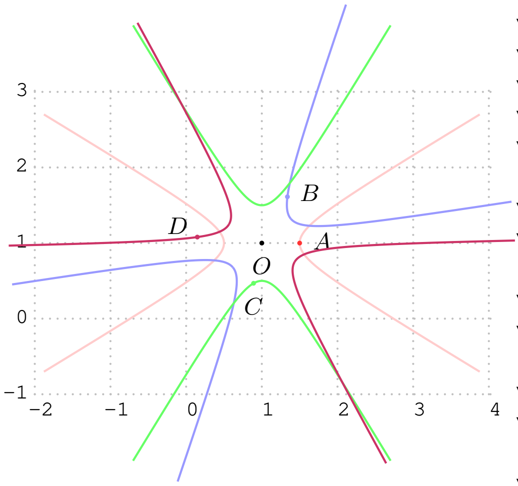
$$\begin{cases} x' = (a\sec\alpha + x_0)\cos\theta - (b\tan\alpha + y_0)\sin\theta = x'_0 + a\sec\alpha\cos\theta - b\tan\alpha\sin\theta \\ y' = (a\sec\alpha + x_0)\sin\theta + (b\tan\alpha + y_0)\cos\theta = y'_0 + a\sec\alpha\sin\theta + b\tan\alpha\cos\theta \end{cases} \tag{18}$$

where the x'_0 and y'_0 are the coordinate of the given center O after rotation. So we get the parametric function of the General Hyperbola with coordinate translation and rotation as following:

$$\begin{cases} x = x_0 + a\sec\alpha\cos\theta - b\tan\alpha\sin\theta \\ y = y_0 + a\sec\alpha\sin\theta + b\tan\alpha\cos\theta \end{cases} \tag{19}$$

The macro `\pstGeneralHyperbola` is used to draw a General Hyperbola with Center O, the half of the real axis a, the half of the imaginary axis b, and the rotation angle θ of the symmetrical axis. The parameter `angleX` is used to truncate the width of the figure, it should be setup from 0 to 90.

`\pstGeneralHyperbola [Options]` $(O)(a,\, b)$ `[`θ`] [angleX]`

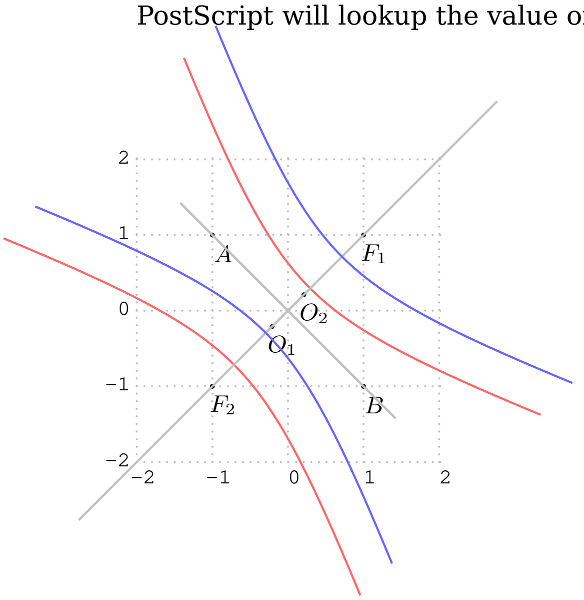



```
\begin{pspicture}[showgrid=true](-2,-1)(4,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\a{0.5}\def\b{0.3}\psset{PointNameSep=0.3}
\pstGeonode[PosAngle=-90](1,1){O}
\pstGeneralHyperbola[linecolor=red!20](O)(\a,\b)[0][80]
\pstGeneralHyperbolaNode[linecolor=red!80,PosAngle=5](O)(\a,\b)
    [0]{0}{A}
\pstGeneralHyperbola[linecolor=blue!40](O)(\a,\b)[40][80]
\pstGeneralHyperbolaNode[linecolor=blue!40,PosAngle=10](O)(\a,\b)
    [40]{40}{B}
\pstGeneralHyperbola[linecolor=green!60](O)(\a,\b)[90][80]
\pstGeneralHyperbolaNode[linecolor=green!60,PosAngle=-90](O)(\a,\
    b)[90]{200}{C}
\pstGeneralHyperbola[linecolor=purple!80](O)(\a,\b)[150][80]
\pstGeneralHyperbolaNode[linecolor=purple!80,PosAngle=150](O)(\a
    ,\b)[150]{50}{D}
\end{pspicture}
```

The Macro \pstGeneralHyperbolaFle is used to define a General Hyperbola with Focus F, directrix line l, and the eccentricity e, where $e > 1$. It just calculate the center O, real radius a, imaginary radius b and the rotation angle θ of the real axis, then you can pass them into macro \pstGeneralHyperbola to draw this hyperbola.

\pstGeneralHyperbolaFle [Options] $\{F\}\{A\}\{B\}\{e\}\{O\}\{Rab\}\{\theta\}$

The output parameter O is a node name to store the center point, its label and symbol can be controlled by the options for PSTricks node, such as PosAngle. The output parameter Rab is a PostScript key to store the pair of real radius and imaginary radius, it just use PSTricks node coordinate to store a pair of value, but not a geometrical point. The output parameter θ is also a PostScript key to store the rotation angle of real axis, when you pass it to \pstGeneralHyperbola, PostScript will lookup the value of this key in current dictionary.

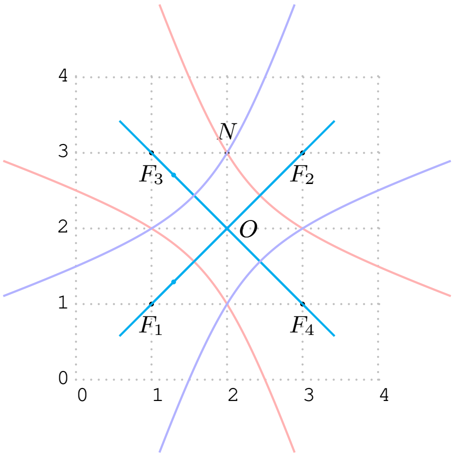



```
\begin{pspicture}[showgrid=true](-2,-2)(2,2)
\psset{unit=1.0cm}\psset{dotscale=0.5}\footnotesize
\psset{CodeFig=true,CodeFigColor=gray!50}\psset{PointSymbol=*}
\pstGeonode[PosAngle=-60](1,1){F_1}
\pstGeonode[PosAngle=-60](-1,-1){F_2}
\pstGeonode[PosAngle=-60](1,-1){B}
\pstGeonode[PosAngle=-60](-1,1){A}
\pstGeneralHyperbolaFle[PosAngle=-60]{F_1}{A}{B}{2.4}{O_1}{R_1}{
    RealAxisRotAngle1}
\pstGeneralHyperbola[linecolor=red!60](O_1)(R_1)[RealAxisRotAngle
    1][65]
\pstGeneralHyperbolaFle[PosAngle=-60]{F_2}{A}{B}{2.4}{O_2}{R_2}{
    RealAxisRotAngle2}
\pstGeneralHyperbola[linecolor=blue!60](O_2)(R_2)[
    RealAxisRotAngle2][65]
\end{pspicture}
```

The Macro \pstGeneralHyperbolaFFN is used to define a General Hyperbola by the given focus nodes F_1, F_2, and one node N on it. It just calculate the center O, major radius a, minor radius b and the rotation angle θ of the major axis, then you can pass them into macro \pstGeneralHyperbola to draw this hyperbola.

\pstGeneralHyperbolaFFN [Options] $\{F_1\}\{F_2\}\{O\}\{Rab\}\{\theta\}$

The output parameter O, the output parameter Rab and the output parameter θ are same with \pstGeneralHyperbolaFle.



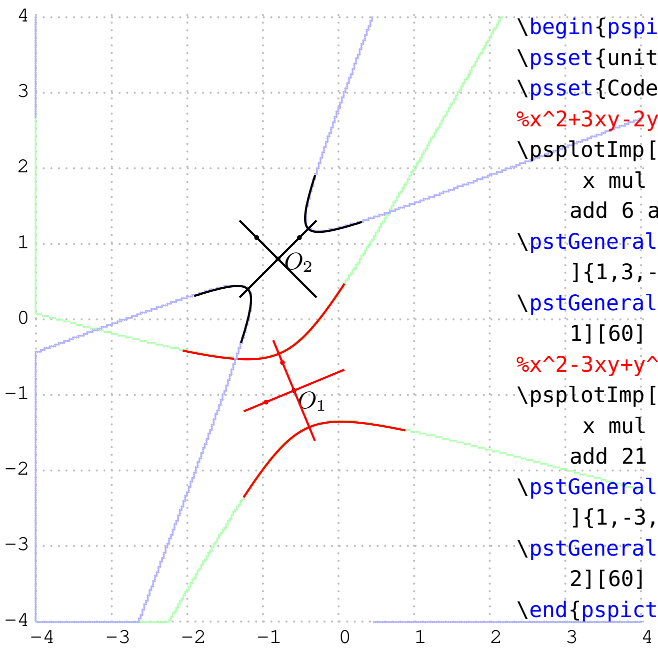

```
\begin{pspicture}[showgrid=true](0,0)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\pstGeonode[PosAngle=-90](1,1){F_1}
\pstGeonode[PosAngle=-90](3,3){F_2}
\pstGeonode[PosAngle=-90](1,3){F_3}
\pstGeonode[PosAngle=-90](3,1){F_4}
\pstGeonode[PosAngle=90](2,3){N}
\pstGeneralHyperbolaFFN[linecolor=red!30,CodeFig=true]{F_1}{F_2}{
    N}{O}{R1}{angle1}
\pstGeneralHyperbola[linecolor=red!30](O)(R1)[angle1][65]
\pstGeneralHyperbolaFFN[linecolor=blue!30,CodeFig=true]{F_3}{F
    _4}{N}{O}{R2}{angle2}
\pstGeneralHyperbola[linecolor=blue!30](O)(R2)[angle2][65]
\end{pspicture}
```

The Macro \pstGeneralHyperbolaCoef is used to define a General Hyperbola by the quadratic curve equation $ax^2 + bxy + cy^2 + dx + ey + f = 0$, it just calculate the center O, real radius a and imaginary radius b and the rotation angle θ of the real axis, then you can pass them into macro \pstGeneralHyperbola to draw this hyperbola. The package pst-func provides macro \psplotImp to draw an implicit defined functions too, but it can't tell you the geometrical elements like as center or radii, and it will take more time to calculate the function value point by point.

\pstGeneralHyperbolaCoef [Options] $\{a,b,c,d,e,f\}\{O\}\{Rab\}\{\theta\}$

The output parameter O, the output parameter Rab and the output parameter θ are same with \pstGeneralHyperbolaFle. They are set to zero if the coeffients are invalid to construct a hyperbola.

In the following example, we use \psplotImp to draw the same hyperbolas, just to check the results given by macros \pstGeneralHyperbolaCoef are correct.

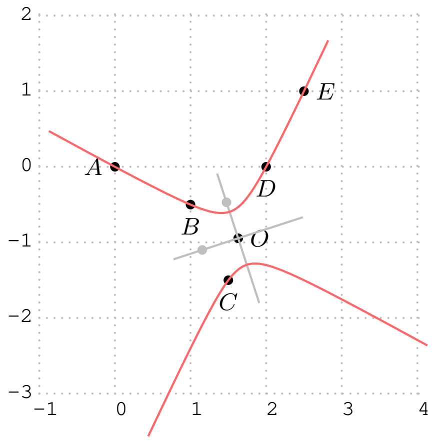



```
\begin{pspicture}[showgrid=true](-4,-4)(4,4)
\psset{unit=0.40cm}\footnotesize\psset{dotscale=0.5}
\psset{CodeFig=true}\psset{PointSymbol=*}
%x^2+3xy-2y^2+10x-5y+6=0
\psplotImp[linecolor=green!30](-10,-10)(10,10){ 1 x dup mul mul 3
     x mul y mul add -2 y dup mul mul add 10 x mul add -5 y mul
    add 6 add }
\pstGeneralHyperbolaCoef[PosAngle=-30,CodeFigColor=red
    ]{1,3,-2,10,-5,6}{O_1}{R_1}{RealAxisRotAngle1}
\pstGeneralHyperbola[linecolor=red](O_1)(R_1)[RealAxisRotAngle
    1][60]
%x^2-3xy+y^2+10x-10y+21=0
\psplotImp[linecolor=blue!30](-10,-10)(10,10){ 1 x dup mul mul -3
     x mul y mul add 1 y dup mul mul add 10 x mul add -10 y mul
    add 21 add }
\pstGeneralHyperbolaCoef[PosAngle=-10,CodeFigColor=black
    ]{1,-3,1,10,-10,21}{O_2}{R_2}{RealAxisRotAngle2}
\pstGeneralHyperbola[linecolor=black](O_2)(R_2)[RealAxisRotAngle
    2][60]
\end{pspicture}
```

The Macro \pstGeneralHyperbolaABCDE is used to define a General Hyperbola by the given five points A, B, C, D, E, it just calculate the center O, real radius a and imaginary radius b and the rotation angle θ of the real axis, then you can pass them into macro \pstGeneralHyperbola to draw this hyperbola.

\pstGeneralHyperbolaABCDE [Options] $\{A\}\{B\}\{C\}\{D\}\{E\}\{O\}\{Rab\}\{\theta\}$

The output parameter O, the output parameter Rab and the output parameter θ are same with \pstGeneralHyperbolaFle. They are set to zero if the points are invalid to construct a hyperbola.

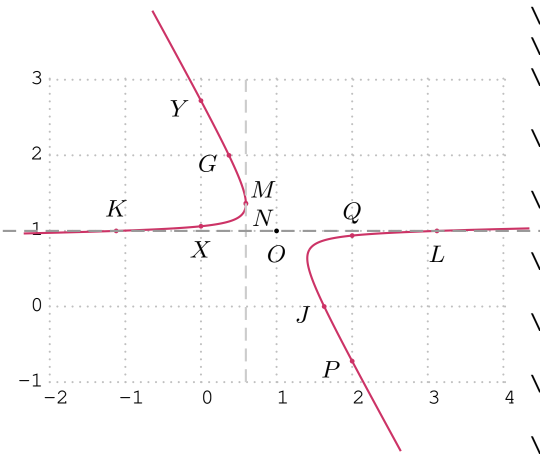



```
\begin{pspicture}[showgrid=true](-1,-3)(4,2)
\psset{unit=0.5cm}\footnotesize\psset{PointSymbol=*}
\psset{CodeFig=true,CodeFigColor=gray!50}
\pstGeonode[PosAngle=180](0,0){A}
\pstGeonode[PosAngle=-90](2,-1){B}
\pstGeonode[PosAngle=-90](3,-3){C}
\pstGeonode[PosAngle=-90](4,0){D}
\pstGeonode[PosAngle=0](5,2){E}
\pstGeneralHyperbolaABCDE[PosAngle=0]{A}{B}{C}{D}{E}{O}{R}{
    RealAxisRotAngle}
\pstGeneralHyperbola[linecolor=red!60](O)(R)[RealAxisRotAngle
    ][80]
\end{pspicture}
```

The macro \pstGeneralHyperbolaNode is used to draw a node whose parameter is the given value t on General Hyperbola, please refer to equation (19). The macro \pstGeneralHyperbolaAbsNode is used to draw the nodes whose abscissa are the given value x_1 on General Hyperbola. The macro \pstGeneralHyperbolaOrdNode is used to draw the nodes whose ordinate are the given value y_1 on General Hyperbola.

Note that \pstGeneralHyperbolaAbsNode and \pstGeneralHyperbolaOrdNode will create two nodes A and B at most time.

\pstGeneralHyperbolaNode [Options] $(O)(a,\ b)$ $[\theta]$ $\{t\}\{A\}$
\pstGeneralHyperbolaAbsNode [Options] $(O)(a,\ b)$ $[\theta]$ $\{x_1\}\{A\}\{B\}$
\pstGeneralHyperbolaOrdNode [Options] $(O)(a,\ b)$ $[\theta]$ $\{y_1\}\{A\}\{B\}$



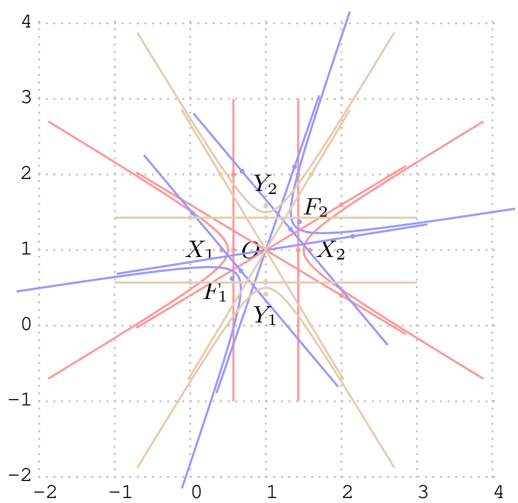

```
\begin{pspicture}[showgrid=true](-2,-1)(4,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\a{0.5}\def\b{0.3}\psset{PointNameSep=0.3}
\pstGeonode[PosAngle=-90](1,1){O}
\pstGeneralHyperbola[linecolor=purple!80](O)(\a,\b)[150][80]
\pstGeneralHyperbolaAbsNode[linecolor=purple!80,PosAngle
    ={200,90}](O)(\a,\b)[150]{2}{P}{Q}
\pstGeneralHyperbolaAbsNode[linecolor=purple!80,PosAngle
    ={-90,200}](O)(\a,\b)[150]{0}{X}{Y}
\pstGeneralHyperbolaAbsNode[linecolor=purple!80,PosAngle
    ={40,-40}](O)(\a,\b)[150]{0.59378}{M}{N}
\pstLine[linestyle=dashed,linecolor=gray
    !40]{0.59378,-1}{0.59378,3}
\pstGeneralHyperbolaOrdNode[linecolor=purple!80,PosAngle
    ={200,90}](O)(\a,\b)[150]{2}{G}{H}
\pstGeneralHyperbolaOrdNode[linecolor=purple!80,PosAngle
    ={-90,200}](O)(\a,\b)[150]{0}{I}{J}
\pstGeneralHyperbolaOrdNode[linecolor=purple!80,PosAngle
    ={90,-90}](O)(\a,\b)[150]{1}{K}{L}
\pstLine[linestyle=dashed,linecolor=gray!80,nodesep=-1.5]{K}{L}
\end{pspicture}
```

The macro \pstGeneralHyperbolaFocusNode is used to find the focus nodes of the General Hyperbola, the macro \pstGeneralHyperbolaVertexNode is used to find the vertex nodes of the General Hyperbola, and the macro \pstGeneralHyperbolaDirectrixLine is used to find the directrix lines of the General Hyperbola.

\pstGeneralHyperbolaFocusNode [Options] $(O)(a,\, b)$ $[\theta]$ $\{F_1\}\{F_2\}$
\pstGeneralHyperbolaVertexNode [Options] $(O)(a,\, b)$ $[\theta]$ $\{V_1\}\{V_2\}$
\pstGeneralHyperbolaDirectrixLine [Options] $(O)(a,\, b)$ $[\theta]$ $\{L_x\}\{L_y\}\{R_x\}\{R_y\}$

Note that you can use the macro \pstGeneralHyperbolaAsymptoteLine to get the asymptote lines, this macro only create one node on each asymptote line, as the other one is the center of the hyperbola.

\pstGeneralHyperbolaAsymptoteLine [Options] $(O)(a,\, b)$ $[\theta]$ $\{L_1\}\{L_2\}$

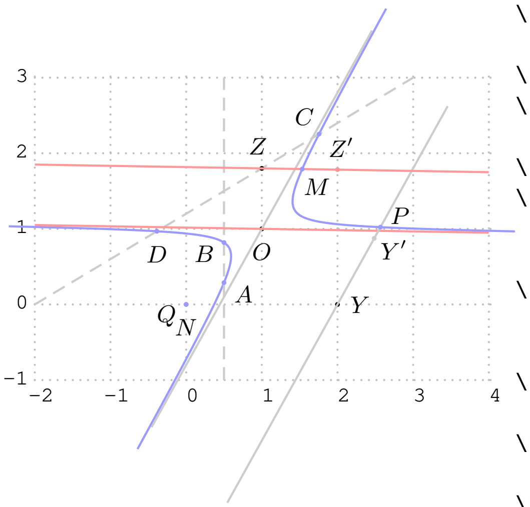



```
\begin{pspicture}[showgrid=true](-2,-2)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\a{0.5}\def\b{0.3}
\pstGeonode[PosAngle=180,PointNameSep=0.2](1,1){O}
\pstGeneralHyperbola[linecolor=red!40](O)(\a,\b)[0][80]
\pstGeneralHyperbolaFocusNode[linecolor=red!40,PointName={X_1,X
    _2},PosAngle={180,0}](O)(\a,\b)[0]{X1}{X2}
\pstGeneralHyperbolaDirectrixLine[linecolor=red!40,nodesepA=-2,
    nodesepB=-1,PointName=none](O)(\a,\b)[0]{Lx}{Ly}{Rx}{Ry}
\pstGeneralHyperbolaAsymptoteLine[linecolor=red!40,nodesepA=-2,
    nodesepB=-1,PointName=none](O)(\a,\b)[0]{L1}{L2}
\pstGeneralHyperbola[linecolor=blue!40](O)(\a,\b)[40][80]
\pstGeneralHyperbolaFocusNode[linecolor=blue!40,PointName={F_1,F
    _2},PosAngle={220,40}](O)(\a,\b)[40]{F1}{F2}
\pstGeneralHyperbolaDirectrixLine[linecolor=blue!40,nodesepA=-2,
    nodesepB=-1,PointName=none](O)(\a,\b)[40]{Dx}{Dy}{Ux}{Uy}
\pstGeneralHyperbolaAsymptoteLine[linecolor=blue!40,nodesepA=-2,
    nodesepB=-1,PointName=none](O)(\a,\b)[40]{S1}{S2}
\pstGeneralHyperbola[linecolor=brown!40](O)(\a,\b)[90][80]
\pstGeneralHyperbolaFocusNode[linecolor=brown!40,PointName={Y_1,Y
    _2},PosAngle={-90,90}](O)(\a,\b)[90]{Y1}{Y2}
\pstGeneralHyperbolaDirectrixLine[linecolor=brown!40,nodesepA=-2,
    nodesepB=-1,PointName=none](O)(\a,\b)[90]{Tx}{Ty}{Sx}{Sy}
\pstGeneralHyperbolaAsymptoteLine[linecolor=brown!40,nodesepA=-2,
    nodesepB=-1,PointName=none](O)(\a,\b)[90]{T1}{T2}
\end{pspicture}
```

The macro \pstGeneralHyperbolaLineInter is used to find the intersections C and D of the general hyperbola and the given line AB.

\pstGeneralHyperbolaLineInter [Options] $(O)(a,\, b)$ $[\theta]$ $\{A\}\{B\}\{C\}\{D\}$

In the following example, the lines YY' and ZZ' are parallel to the asymptote of the hyperbola, so there are only one intersection M and P for each line, and the second node N and Q are put at the origin.

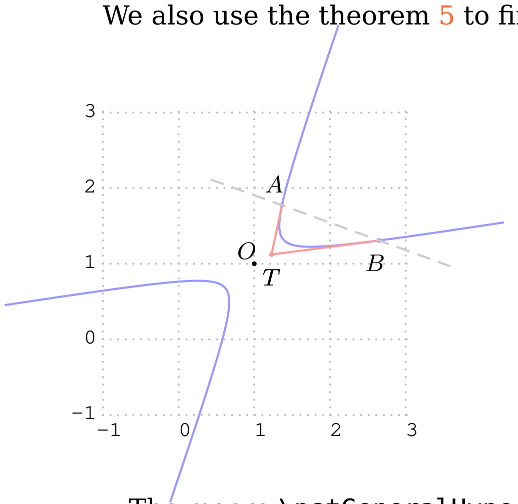



```
\begin{pspicture}[showgrid=true](-2,-1)(4,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\a{0.5}\def\b{0.3}\psset{PointNameSep=0.3}
\pstGeonode[PosAngle=-90](1,1){O}
\pstGeneralHyperbola[linecolor=blue!40](O)(\a,\b)[30][80]
\pstLine[linestyle=dashed,linecolor=gray!40]{0.5,-1}{0.5,3}
\pstGeneralHyperbolaLineInter[linecolor=blue!40,PosAngle
    ={-30,210}](O)(\a,\b)[30]{0.5,-1}{0.5,3}{A}{B}
\pstLine[linestyle=dashed,linecolor=gray!40]{-2,0}{3,3}
\pstGeneralHyperbolaLineInter[linecolor=blue!40,PosAngle
    ={130,-90}](O)(\a,\b)[30]{-2,0}{3,3}{C}{D}
\pstGeonode[PosAngle={0,100}](2,0){Y}(1,1.8){Z}
\pstLineAA[nodesepA=-3,nodesepB=-2,linecolor=gray!40,PointName=
    none,PointSymbol=none]{O}{\b\space \a\space div 1 atan 30
    add}{U}
\pstLineAA[nodesepA=-3,nodesepB=-2,linecolor=red!40,PointName=
    none,PointSymbol=none]{O}{\b\space \a\space div neg 1 atan
    30 add}{V}
\pstLineAA[nodesepA=-3,nodesepB=-2,linecolor=gray!40,PosAngle
    =-30]{Y}{\b\space \a\space div 1 atan 30 add}{Y'}
\pstLineAA[nodesepA=-3,nodesepB=-2,linecolor=red!40,PosAngle=80]{
    Z}{\b\space \a\space div neg 1 atan 30 add}{Z'}
\pstGeneralHyperbolaLineInter[linecolor=blue!40,PosAngle
    ={-50,-90}](O)(\a,\b)[30]{Z}{Z'}{M}{N}
\pstGeneralHyperbolaLineInter[linecolor=blue!40,PosAngle
    ={30,210}](O)(\a,\b)[30]{Y}{Y'}{P}{Q}
\end{pspicture}
```

The macro \pstGeneralHyperbolaPolarNode is used to find the polar point T of chord AB on the general hyperbola.

\pstGeneralHyperbolaPolarNode [Options] $(O)(a,\,b)$ $[\theta]$ $\{A\}\{B\}\{T\}$

We also use the theorem 5 to find the polar point T of chord AB:

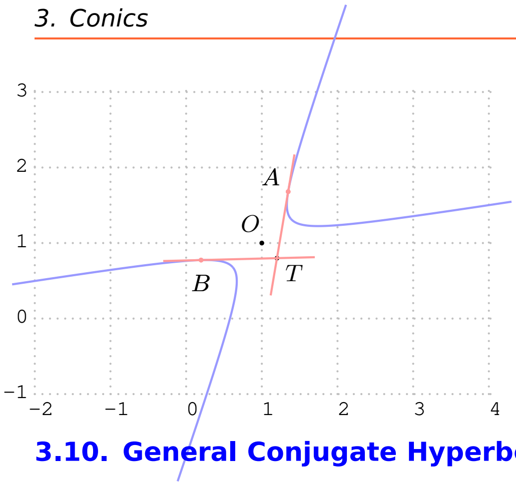



```
\begin{pspicture}[showgrid=true](-1,-1)(3,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\a{0.5}\def\b{0.3}\psset{PointNameSep=0.3}
\pstGeonode[PosAngle=120,PointNameSep=0.2](1,1){O}
\pstGeneralHyperbola[linecolor=blue!40](O)(\a,\b)[40][80]
\pstGeneralHyperbolaNode[linecolor=blue!40,PosAngle=110](O)(\a,\b
    )[40]{50}{A}
\pstGeneralHyperbolaNode[linecolor=blue!40,PosAngle=-100](O)(\a,\
    b)[40]{-70}{B}
\pstGeneralHyperbolaPolarNode[linecolor=red!40,PosAngle=-90](O)(\
    a,\b)[40]{A}{B}{T}
\pstLine[linestyle=dashed,linecolor=gray!40,nodesep=-1]{A}{B}
\end{pspicture}
```

The macro \pstGeneralHyperbolaTangentNode is used to find the tangent point A and B of point T outside of the general hyperbola.

\pstGeneralHyperbolaTangentNode [Options] $(O)(a,\,b)$ $[\theta]$ $\{T\}\{A\}\{B\}$

We also use the theorem 6 to find the tangent points A and B of T.




```
\begin{pspicture}[showgrid=true](-2,-1)(4,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\a{0.5}\def\b{0.3}\psset{PointNameSep=0.3}
\pstGeonode[PosAngle=120](1,1){O}
\pstGeneralHyperbola[linecolor=blue!40](O)(\a,\b)[40][80]
\pstGeonode[PosAngle=-40](1.2,0.8){T}
\pstGeneralHyperbolaTangentNode[linecolor=red!40,PosAngle
    ={140,-90},nodesep=-0.5](O)(\a,\b)[40]{T}{A}{B}
\end{pspicture}
```

3.10. General Conjugate Hyperbola

The General Conjugate Hyperbola H with coordinate translation and rotation is defined by center O, the half of the real axis a, the half of the imaginary axis b, and the rotation angle θ of the principal axis. The equation can be got from the parametric function of the Standard Conjugate Hyperbola equation (17), using the rotation transform formula (3), then we have

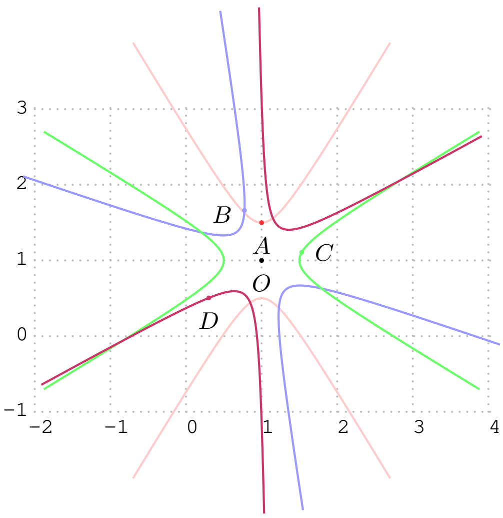
$$\begin{cases} x' = (b\tan\alpha + x_0)\cos\theta - (a\sec\alpha + y_0)\sin\theta = x'_0 + b\tan\alpha\cos\theta - a\sec\alpha\sin\theta \\ y' = (b\tan\alpha + x_0)\sin\theta + (a\sec\alpha + y_0)\cos\theta = y'_0 + b\tan\alpha\sin\theta + a\sec\alpha\cos\theta \end{cases} \tag{20}$$

where the x'_0 and y'_0 are the coordinate of the given center O after rotation. So we get the parametric function of the General Conjugate Hyperbola with coordinate translation and rotation as following:

$$\begin{cases} x = x_0 + b\tan\alpha\cos\theta - a\sec\alpha\sin\theta \\ y = y_0 + b\tan\alpha\sin\theta + a\sec\alpha\cos\theta \end{cases} \tag{21}$$

The macro `\pstGeneralIHyperbola` is used to draw a General Conjugate Hyperbola with Center O, the half of the real axis a, the half of the imaginary axis b, and the rotation angle θ of the symmetrical axis. The parameter `angleY` is used to truncate the height of the figure, it should be setup from 0 to 90.

`\pstGeneralIHyperbola [Options] (`O`)(`a, b`) [`θ`] [angleY]`



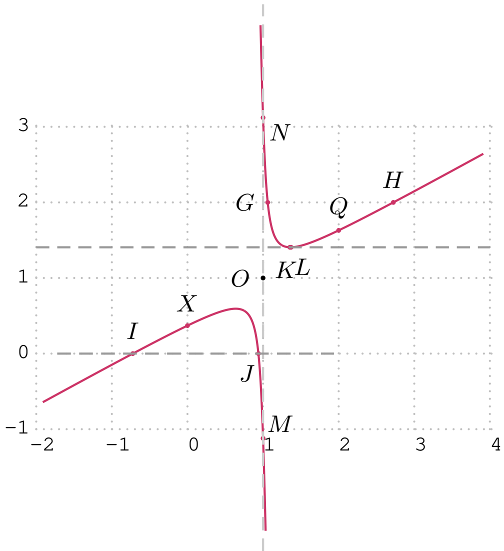

```
\begin{pspicture}[showgrid=true](-2,-1)(4,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\a{0.5}\def\b{0.3}\psset{PointNameSep=0.3}
\pstGeonode[PosAngle=-90](1,1){O}
\pstGeneralIHyperbola[linecolor=red!20](O)(\a,\b)[0][80]
\pstGeneralIHyperbolaNode[linecolor=red!80,PosAngle=-90](O)(\a,\b
    )[0]{0}{A}
\pstGeneralIHyperbola[linecolor=blue!40](O)(\a,\b)[40][80]
\pstGeneralIHyperbolaNode[linecolor=blue!40,PosAngle=190](O)(\a,\
    b)[40]{40}{B}
\pstGeneralIHyperbola[linecolor=green!60](O)(\a,\b)[90][80]
\pstGeneralIHyperbolaNode[linecolor=green!60,PosAngle=0](O)(\a,\b
    )[90]{200}{C}
\pstGeneralIHyperbola[linecolor=purple!80](O)(\a,\b)[150][80]
\pstGeneralIHyperbolaNode[linecolor=purple!80,PosAngle=-90](O)(\a
    ,\b)[150]{50}{D}
\end{pspicture}
```

The macro `\pstGeneralIHyperbolaNode` is used to draw a node whose parameter is the given value t on General Conjugate Hyperbola, please refer to equation (21).

The macro `\pstGeneralIHyperbolaAbsNode` is used to draw the nodes whose abscissa are the given value x_1 on General Conjugate Hyperbola. The macro `\pstGeneralIHyperbolaOrdNode` is used to draw the nodes whose ordinate are the given value y_1 on General Conjugate Hyperbola.

Note that `\pstGeneralIHyperbolaAbsNode` and `\pstGeneralIHyperbolaOrdNode` will create two nodes A and B at most time.

`\pstGeneralIHyperbolaNode` `[Options]` (O)(a, b) `[`θ`]` {t}{A}
`\pstGeneralIHyperbolaAbsNode` `[Options]` (O)(a, b) `[`θ`]` {x_1}{A}{B}
`\pstGeneralIHyperbolaOrdNode` `[Options]` (O)(a, b) `[`θ`]` {y_1}{A}{B}



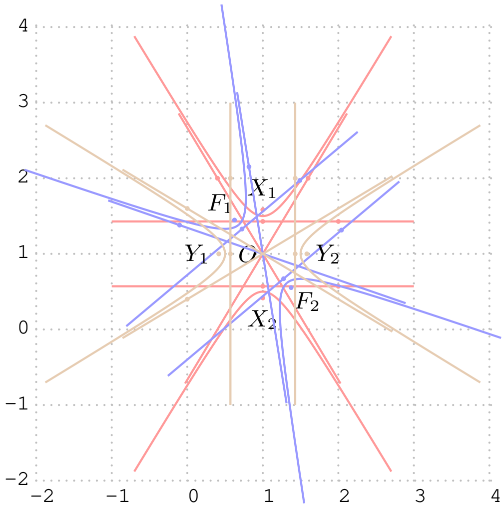

```
\begin{pspicture}[showgrid=true](-2,-1)(4,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\a{0.5}\def\b{0.3}\psset{PointNameSep=0.3}
\pstGeonode[PosAngle=180](1,1){O}
\pstGeneralIHyperbola[linecolor=purple!80](O)(\a,\b)[150][80]
\pstGeneralIHyperbolaAbsNode[linecolor=purple!80,PosAngle
    ={200,90}](O)(\a,\b)[150]{2}{P}{Q}
\pstGeneralIHyperbolaAbsNode[linecolor=purple!80,PosAngle
    ={90,200}](O)(\a,\b)[150]{0}{X}{Y}
\pstGeneralIHyperbolaAbsNode[linecolor=purple!80,PosAngle
    ={40,-40}](O)(\a,\b)[150]{1}{M}{N}
\pstLine[linestyle=dashed,linecolor=gray!40,nodesep=-1.5]{M}{N}
\pstGeneralIHyperbolaOrdNode[linecolor=purple!80,PosAngle
    ={180,90}](O)(\a,\b)[150]{2}{G}{H}
\pstGeneralIHyperbolaOrdNode[linecolor=purple!80,PosAngle
    ={90,240}](O)(\a,\b)[150]{0}{I}{J}
\pstGeneralIHyperbolaOrdNode[linecolor=purple!80,PosAngle
    ={-100,-60}](O)(\a,\b)[150]{1.4063}{K}{L}
\pstLine[linestyle=dashed,linecolor=gray!80,nodesep=-1]{I}{J}
\pstLine[linestyle=dashed,linecolor=gray!80,nodesep
    =-1]{-1,1.4063}{3,1.4063}
\end{pspicture}
```

The macro `\pstGeneralIHyperbolaFocusNode` is used to find the focus nodes of the General Conjugate Hyperbola, the macro `\pstGeneralIHyperbolaVertexNode` is used to find the vertex nodes of the General Conjugate Hyperbola, and the macro `\pstGeneralIHyperbolaDirectrixLine` is used to find the directrix lines of the General Conjugate Hyperbola.

`\pstGeneralIHyperbolaFocusNode` `[Options]` (O)(a, b) `[`θ`]` {F_1}{F_2}
`\pstGeneralIHyperbolaVertexNode` `[Options]` (O)(a, b) `[`θ`]` {V_1}{V_2}
`\pstGeneralIHyperbolaDirectrixLine` `[Options]` (O)(a, b) `[`θ`]` {L_x}{L_y}{R_x}{R_y}

Note that you can use the macro `\pstGeneralIHyperbolaAsymptoteLine` to get the asymptote lines, this macro only create one node on each asymptote line, as the other one is the center of the hyperbola.

`\pstGeneralHyperbolaAsymptoteLine` `[Options]` (O)(a, b) `[`θ`]` {L_1}{L_2}

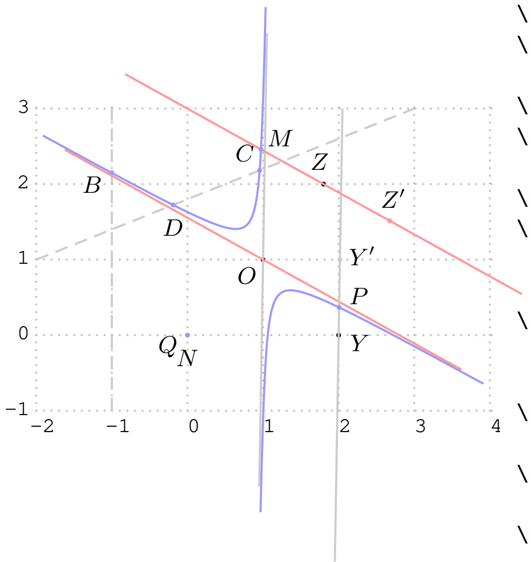



```
\begin{pspicture}[showgrid=true](-2,-2)(4,4)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\a{0.5}\def\b{0.3}
\pstGeonode[PosAngle=180,PointNameSep=0.2](1,1){O}
\pstGeneralIHyperbola[linecolor=red!40](O)(\a,\b)[0][80]
\pstGeneralIHyperbolaFocusNode[linecolor=red!40,PointName={X_1,X
    _2},PosAngle={90,-90}](O)(\a,\b)[0]{X1}{X2}
\pstGeneralIHyperbolaDirectrixLine[linecolor=red!40,nodesepA=-2,
    nodesepB=-1,PointName=none](O)(\a,\b)[0]{Lx}{Ly}{Rx}{Ry}
\pstGeneralIHyperbolaAsymptoteLine[linecolor=red!40,nodesepA=-2,
    nodesepB=-1,PointName=none](O)(\a,\b)[0]{T1}{T2}
\pstGeneralIHyperbola[linecolor=blue!40](O)(\a,\b)[40][80]
\pstGeneralIHyperbolaFocusNode[linecolor=blue!40,PointName={F_1,F
    _2},PosAngle={130,-40}](O)(\a,\b)[40]{F1}{F2}
\pstGeneralIHyperbolaDirectrixLine[linecolor=blue!40,nodesepA=-2,
    nodesepB=-1,PointName=none](O)(\a,\b)[40]{Dx}{Dy}{Ux}{Uy}
\pstGeneralIHyperbolaAsymptoteLine[linecolor=blue!40,nodesepA=-2,
    nodesepB=-1,PointName=none](O)(\a,\b)[40]{S1}{S2}
\pstGeneralIHyperbola[linecolor=brown!40](O)(\a,\b)[90][80]
\pstGeneralIHyperbolaFocusNode[linecolor=brown!40,PointName={Y_1,
    Y_2},PosAngle={180,0}](O)(\a,\b)[90]{Y1}{Y2}
\pstGeneralIHyperbolaDirectrixLine[linecolor=brown!40,nodesepA
    =-2,nodesepB=-1,PointName=none](O)(\a,\b)[90]{Tx}{Ty}{Sx}{Sy
    }
\pstGeneralIHyperbolaAsymptoteLine[linecolor=brown!40,nodesepA
    =-2,nodesepB=-1,PointName=none](O)(\a,\b)[90]{R1}{R2}
\end{pspicture}
```

The macro `\pstGeneralIHyperbolaLineInter` is used to find the intersections C and D of the general conjugate hyperbola and the given line AB.

`\pstGeneralIHyperbolaLineInter [Options] (`O`)(`a, b`) [`θ`] {`A`}{`B`}{`C`}{`D`}`

In the following example, the lines YY' and ZZ' are parallel to the asymptote of the hyperbola, so there are only one intersection M and P for each line, and the second node N and Q are put at the origin.

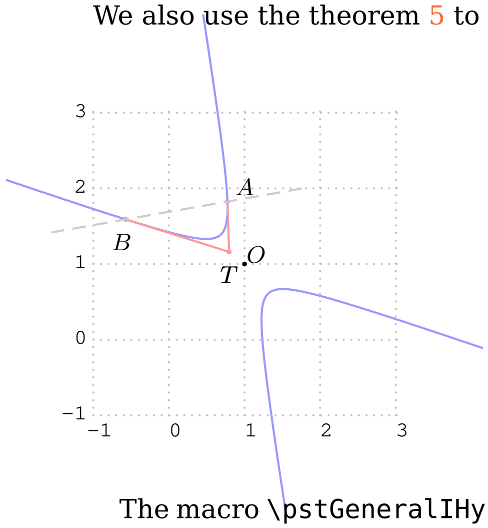



```
\begin{pspicture}[showgrid=true](-2,-1)(4,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\a{0.5}\def\b{0.3}\psset{PointNameSep=0.3}
\pstGeonode[PosAngle=225](1,1){O}
\pstGeneralIHyperbola[linecolor=blue!40](O)(\a,\b)[30][80]
\pstLine[linestyle=dashed,linecolor=gray!40]{-1,-1}{-1,3}
\pstGeneralIHyperbolaLineInter[linecolor=blue!40,PosAngle
    ={-30,210}](O)(\a,\b)[30]{-1,-1}{-1,3}{A}{B}
\pstLine[linestyle=dashed,linecolor=gray!40]{-2,1}{3,3}
\pstGeneralIHyperbolaLineInter[linecolor=blue!40,PosAngle
    ={130,-90}](O)(\a,\b)[30]{-2,1}{3,3}{C}{D}
\pstGeonode[PosAngle={-20,100}](2,0){Y}(1.8,2){Z}
\pstLineAA[nodesepA=-3,nodesepB=-2,linecolor=gray!40,PointName=
    none,PointSymbol=none]{O}{\a\space \b\space div 1 atan 30
    add}{U}
\pstLineAA[nodesepA=-3,nodesepB=-2,linecolor=red!40,PointName=
    none,PointSymbol=none]{O}{\a\space \b\space div neg 1 atan
    30 add}{V}
\pstLineAA[nodesepA=-3,nodesepB=-2,linecolor=gray!40,PosAngle
    =10]{Y}{\a\space \b\space div 1 atan 30 add}{Y'}
\pstLineAA[nodesepA=-3,nodesepB=-2,linecolor=red!40,PosAngle=80]{
    Z}{\a\space \b\space div neg 1 atan 30 add}{Z'}
\pstGeneralIHyperbolaLineInter[linecolor=blue!40,PosAngle
    ={30,-90}](O)(\a,\b)[30]{Z}{Z'}{M}{N}
\pstGeneralIHyperbolaLineInter[linecolor=blue!40,PosAngle
    ={30,210}](O)(\a,\b)[30]{Y}{Y'}{P}{Q}
\end{pspicture}
```

The macro \pstGeneralIHyperbolaPolarNode is used to find the polar point T of chord AB on the general conjugate hyperbola.

\pstGeneralIHyperbolaPolarNode [Options] $(O)(a,\,b)$ [θ] $\{A\}\{B\}\{T\}$

We also use the theorem 5 to find the polar point T of chord AB:

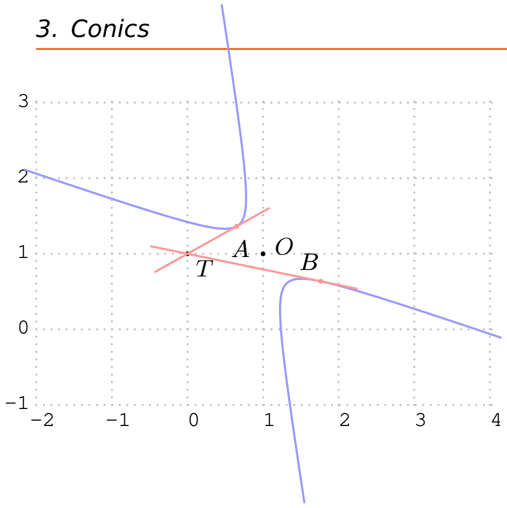



```
\begin{pspicture}[showgrid=true](-1,-1)(3,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\a{0.5}\def\b{0.3}\psset{PointNameSep=0.3}
\pstGeonode[PosAngle=40,PointNameSep=0.2](1,1){O}
\pstGeneralIHyperbola[linecolor=blue!40](O)(\a,\b)[40][80]
\pstGeneralIHyperbolaNode[linecolor=blue!40,PosAngle=40](O)(\a,\b
    )[40]{50}{A}
\pstGeneralIHyperbolaNode[linecolor=blue!40,PosAngle=-100](O)(\a
    ,\b)[40]{-70}{B}
\pstGeneralIHyperbolaPolarNode[linecolor=red!40,PosAngle=-90](O)
    (\a,\b)[40]{A}{B}{T}
\pstLine[linestyle=dashed,linecolor=gray!40,nodesep=-1]{A}{B}
\end{pspicture}
```

The macro \pstGeneralIHyperbolaTangentNode is used to find the tangent point A and B of point T outside of the general conjugate hyperbola.

\pstGeneralIHyperbolaTangentNode [Options] $(O)(a,\,b)$ [θ] $\{T\}\{A\}\{B\}$

We also use the theorem 6 to find the tangent points A and B of T.




```
\begin{pspicture}[showgrid=true](-2,-1)(4,3)
\psset{dotscale=0.5}\psset{PointSymbol=*}\footnotesize
\def\a{0.5}\def\b{0.3}\psset{PointNameSep=0.3}
\pstGeonode[PosAngle=20](1,1){O}
\pstGeneralIHyperbola[linecolor=blue!40](O)(\a,\b)[40][80]
\pstGeonode[PosAngle=-40](0,1){T}
\pstGeneralIHyperbolaTangentNode[linecolor=red!40,PosAngle
    ={-80,120},nodesep=-0.5](O)(\a,\b)[40]{T}{A}{B}
\end{pspicture}
```

3.11. General Conics

A General Conic is defined by the quadratic curve equation

$$ax^2 + bxy + cy^2 + dx + ey + f = 0 \tag{22}$$

it can be reduced to an ellipse, hyperbola or parabola after translation and rotation.

In the previous sections, we use the macros `\pstGeneralEllipseCoef`, `\pstGeneralHyperbolaCoef`, `\pstGeneralParabolaCoef` to define the general ellipse, general hyperbola and general parabola from the quadratic curve equation (22). Here we provide the macros `\pstGeneralConicEquation`, `\pstGeneralEllipseEquation`, `\pstGeneralHyperbolaEquation` and `\pstGeneralParabolaEquation` to get the coefficients a, b, c, d, e, f of their quadratic curve equation (22).

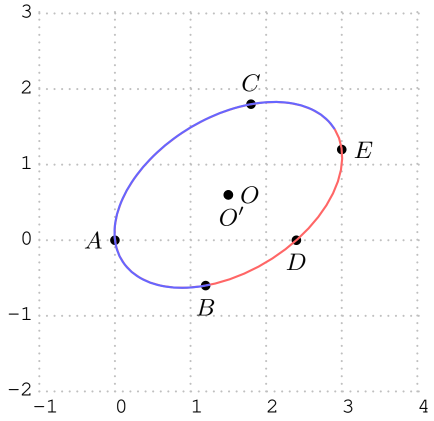
`\pstGeneralConicEquation [Options]` {*A*}{*B*}{*C*}{*D*}{*E*}{*F*}{*a,b,c,d,e,f*}
`\pstGeneralEllipseEquation [Options]` (*O*)(*a,b*) [θ] {*a,b,c,d,e,f*}
`\pstGeneralHyperbolaEquation [Options]` (*O*)(*a,b*) [θ] {*a,b,c,d,e,f*}
`\pstGeneralParabolaEquation [Options]` (*O*){*p*} [θ] {*a,b,c,d,e,f*}

The macro `\pstGeneralConicEquation` take five points A, B, C, D, E as input parameter; the macro `\pstGeneralEllipseEquation` take the general ellipse parameter O, a, b, θ as input parameter; the macro `\pstGeneralHyperbolaEquation` take the general hyperbola parameter O, a, b, θ as input parameter; the macro `\pstGeneralParabolaEquation` take the general parabola parameter O, p, θ as input parameter. They all output the six coefficients a, b, c, d, e, f of the quadratic curve equation.

Note that the output coefficients a, b, c, d, e, f are the variables in `PostScript` level, they will be dumped out in `PostScript`'s console when you set `CodeFig` to `true`. You can pass them into the macros where need these six coefficients. In order to prevent the name conflict in `PostScript` level, You should setup the name of the coefficients more than three letters, for example,

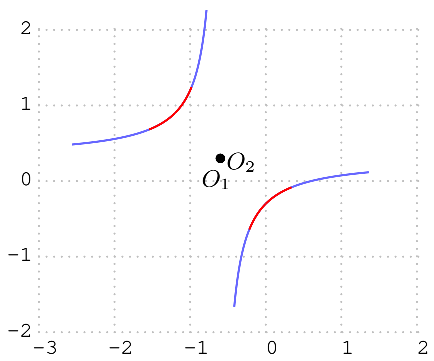
```
% calculate the coefficients CoefA,CoefB,...,CoefF and CoefA',CoefB',...,CoefF'
\pstGeneralConicEquation{A}{B}{C}{D}{E}{CoefA,CoefB,CoefC,CoefD,CoefE,CoefF}
\pstGeneralConicEquation{A'}{B'}{C'}{D'}{E'}{CoefA',CoefB',CoefC',CoefD',CoefE',CoefF'}
% then use them to find the intersections
\pstGeneralConicInter{CoefA,CoefB,CoefC,CoefD,CoefE,CoefF}{CoefA',CoefB',CoefC',CoefD',CoefE',CoefF
    '}{I}{J}{I'}{J'}
```

The following example output the coefficients a, b, c, d, e, f for the ellipse defined by five points, and then draw the ellipse using these coefficients to check the result.



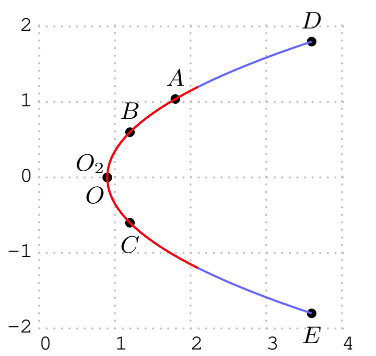

```
\begin{pspicture}[showgrid=true](-1,-2)(4,3)
\psset{unit=0.6cm}\footnotesize\psset{PointSymbol=*}
\psset{CodeFig=false,CodeFigColor=gray!50}
\pstGeonode[PosAngle={180,-90,90,-90,0}](0,0){A}(2,-1){B}(3,3){C}(4,0)
    {D}(5,2){E}
\pstGeneralEllipseABCDE[PosAngle=0]{A}{B}{C}{D}{E}{O}{R}{RotAngle}
\pstGeneralEllipse[linecolor=red!60](O)(R)[RotAngle]
\pstGeneralEllipseEquation(O)(R)[RotAngle]{a,b,c,d,e,f}
\pstGeneralEllipseCoef[PosAngle=-80,CodeFigColor=blue!50]{a,b,c,d,e,f
    }{O'}{R'}{RotAngle1}
\pstGeneralEllipse[linecolor=blue!60](O')(R')[RotAngle1][0][240]
\end{pspicture}
```

The following example output the coefficients a, b, c, d, e, f for the hyperbola $2xy - x + 2y + 1 = 0$, just for example, and then draw the hyperbola using these coefficients to check the result.




```
\begin{pspicture}[showgrid=true](-3,-2)(2,2)
\psset{unit=0.6cm}\footnotesize\psset{PointSymbol=*}
\psset{CodeFig=false,CodeFigColor=gray!50}
% 2xy-x+2y+1=0
\pstGeneralHyperbolaCoef[PosAngle=-100,CodeFigColor=red
    !50]{0,2,0,-1,2,1}{O_1}{R_1}{RotAngle}
\pstGeneralHyperbola[linecolor=blue!60](O_1)(R_1)[RotAngle][56]
\pstGeneralHyperbolaEquation(O_1)(R_1)[RotAngle]{a,b,c,d,e,f}
\pstGeneralHyperbolaCoef[PosAngle=-10,CodeFigColor=red!60]{a,b,c,d,e,f
    }{O_2}{R_2}{RotAngle1}
\pstGeneralHyperbola[linecolor=red](O_2)(R_2)[RotAngle1][25]
\end{pspicture}
```

The following example output the coefficients a, b, c, d, e, f for the parabola defined by five points, and then draw the parabola using these coefficients to check the result.




```
\begin{pspicture}[showgrid=true](0,-2)(4,2)
\psset{unit=0.6cm}\footnotesize\psset{PointSymbol=*}
\psset{CodeFig=false,CodeFigColor=gray!50}
% five points from y^2-2x+3=0
\pstGeonode[PosAngle={90,90,-90,90,-90}](3,1.73205){A}(2,1){B}(2,-1){C
    }(6,3){D}(6,-3){E}
\pstGeneralParabolaABCDE[PosAngle=235]{A}{B}{C}{D}{E}{O}{P}{RotAngle}
\pstGeneralParabola[linecolor=blue!60](O){P}[RotAngle]{-3}{3}
\pstGeneralParabolaEquation(O){P}[RotAngle]{a,b,c,d,e,f}
\pstGeneralParabolaCoef[PosAngle=140,CodeFigColor=black!60]{a,b,c,d,e,
    f}{O_2}{P2}{RotAngle1}
\pstGeneralParabola[linecolor=red](O_2){P2}[RotAngle1]{-2}{2}
\end{pspicture}
```

Here are some macros to find the intersections with the quadratic curve equation (22).

```
\pstGeneralConicLineInter [Options] {A}{B}{a,b,c,d,e,f}{C}{D}
\pstGeneralConicCircleInter [Options] {O}{A}{a,b,c,d,e,f}{C}{D}{E}{F}
\pstGeneralConicEllipseInter [Options] (O)(m,n){a,b,c,d,e,f}{C}{D}{E}{F}
\pstGeneralConicHyperbolaInter [Options] (O)(m,n){a,b,c,d,e,f}{C}{D}{E}{F}
\pstGeneralConicIHyperbolaInter [Options] (O)(m,n){a,b,c,d,e,f}{C}{D}{E}{F}
\pstGeneralConicParabolaInter [Options] (O){p}{a,b,c,d,e,f}{C}{D}{E}{F}
\pstGeneralConicIParabolaInter [Options] (O){p}{a,b,c,d,e,f}{C}{D}{E}{F}
\pstGeneralConicInter{a,b,c,d,e,f}{a',b',c',d',e',f'}{C}{D}{E}{F}
```

The macro `\pstGeneralConicLineInter` is used to find the intersections C, D of the quadratic curve equation (22) and the given line AB.

The macro `\pstGeneralConicCircleInter` is used to find the intersections C, D, E, F of the quadratic curve equation (22) and the given circle $(x - x_A)^2 + (y - y_A)^2 = |OA|^2$.

The macro `\pstGeneralConicEllipseInter` is used to find the intersections C, D, E, F of the quadratic curve equation (22) and the given ellipse $\dfrac{(x - x_O)^2}{m^2} + \dfrac{(y - y_O)^2}{n^2} = 1$.

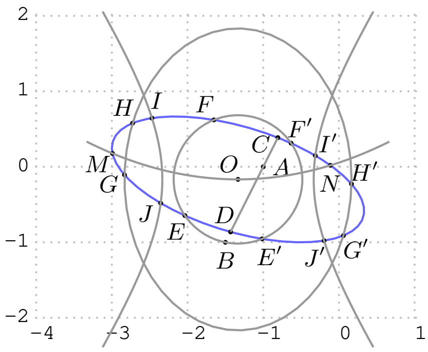
The macro `\pstGeneralConicHyperbolaInter` is used to find the intersections C, D, E, F of the quadratic curve equation (22) and the given hyperbola $\dfrac{(x - x_O)^2}{m^2} - \dfrac{(y - y_O)^2}{n^2} = 1$.

The macro `\pstGeneralConicIHyperbolaInter` is used to find the intersections C, D, E, F of the quadratic curve equation (22) and the given conjugate hyperbola $\dfrac{(y - y_O)^2}{m^2} - \dfrac{(x - x_O)^2}{n^2} = 1$.

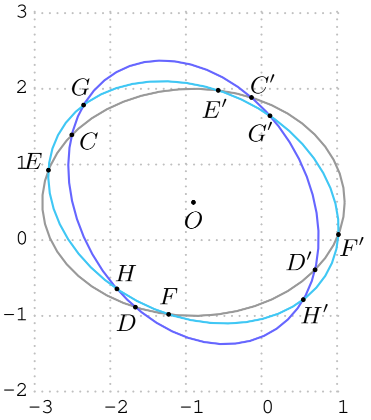
The macro `\pstGeneralConicParabolaInter` is used to find the intersections C, D, E, F of the quadratic curve equation (22) and the given parabola $(x - x_O)^2 = 2p(y - y_O)$.

The macro `\pstGeneralConicIParabolaInter` is used to find the intersections C, D, E, F of the quadratic curve equation (22) and the given conjugate parabola $(y - y_O)^2 = 2p(x - x_O)$.

The macro `\pstGeneralConicInter` is used to find the intersections C, D, E, F of the quadratic curve equation (22) and the other one.




```
\begin{pspicture}[showgrid=true](-4,-2)(1,2)\footnotesize
\psset{unit=0.5cm,PointSymbol=*,dotscale=0.5,PointNameSep=0.24cm}
\pstGeonode[PosAngle={0,-90}](-2,0){A}(-3,-2){B}
\pstGeneralEllipseCoef[PosAngle=120]{1,2,4,6,8,1}{O}{R}{RotAngle}
\pstGeneralEllipse[linecolor=blue!60](O)(R)[RotAngle]
\pstGeneralConicLineInter[PosAngle={200,110}]{A}{B}{1,2,4,6,8,1}{C}{D}
\pstGeneralConicCircleInter[PosAngle={-120,120,-65,60}]{O}{B
    }{1,2,4,6,8,1}{E}{F}{E'}{F'}
\pstGeneralConicEllipseInter[PosAngle={210,120,-45,45}](O)(3,4)
    {1,2,4,6,8,1}{G}{H}{G'}{H'}
\pstGeneralConicHyperbolaInter[PosAngle={75,215,45,240}](O)(2,3)
    {1,2,4,6,8,1}{I}{J}{I'}{J'}
\pstGeneralConicParabolaInter[PosAngle={215,-90},PointName={M,N,none},
    PointSymbol={*,*,none}](O){8}{1,2,4,6,8,1}{M}{N}{M'}{N'}
\pstLineAB[linecolor=gray!80]{C}{D}
\pstCircleOA[linecolor=gray!80]{O}{B}
\pstEllipse[linecolor=gray!80](O)(3,4)
\pstHyperbola[linecolor=gray!80](O)(2,3)[56]
\pstParabola[linecolor=gray!80](O){8}{-4}{4}
\end{pspicture}
```

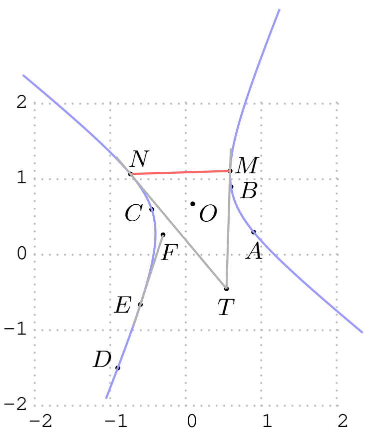



```
\begin{pspicture}[showgrid=true](-3,-2)(1,3)\footnotesize
\psset{unit=0.5cm,PointSymbol=*,dotscale=0.5,PointNameSep=0.24cm}
\pstGeonode[PosAngle={-90,-90}](-1.8,1){O}
\pstEllipse[linecolor=gray!80](O)(4,3)
\pstGeneralEllipse[linecolor=blue!60](O)(3,4)[32]
\pstGeneralEllipse[linecolor=cyan!60](O)(3,4)[65]
\pstGeneralEllipseEquation[CodeFig=false](O)(3,4)[32]{Ea,Eb,Ec,Ed,Ee,
    Ef}
\pstGeneralEllipseEquation[CodeFig=false](O)(3,4)[65]{Ea',Eb',Ec',Ed',
    Ee',Ef'}
\pstGeneralConicEllipseInter[PosAngle={-20,240,50,150}](O)(4,3){Ea,Eb,
    Ec,Ed,Ee,Ef}{C}{D}{C'}{D'}
\pstGeneralConicEllipseInter[PosAngle={150,90,-100,-40}](O)(4,3){Ea',
    Eb',Ec',Ed',Ee',Ef'}{E}{F}{E'}{F'}
\pstGeneralConicInter[PosAngle={100,60,240,-50}]{Ea,Eb,Ec,Ed,Ee,Ef}{Ea
    ',Eb',Ec',Ed',Ee',Ef'}{G}{H}{G'}{H'}
\end{pspicture}
```

Here are some macros to find the tangent line or tangent chord of the quadratic curve equation (22).

`\pstGeneralConicTangentLine [Options]` $\{A\}\{a,b,c,d,e,f\}\{B\}$
`\pstGeneralConicTangentChord [Options]` $\{T\}\{a,b,c,d,e,f\}\{A\}\{B\}$

The macro `\pstGeneralConicTangentLine` find the Tangent Line of the quadratic curve through the point A which is lie on the curve, output the other node B on the tangent line. The macro `\pstGeneralConicTangentChord` find the Tangent Chord of the quadratic curve through the point T which is not lie on the curve, output the tangent point A and B on the curve.




```
\begin{pspicture}[showgrid=true](-2,-2)(2,2)\footnotesize
\psset{unit=0.3cm,PointSymbol=*,dotscale=0.5,PointNameSep=0.24cm}
\pstGeonode[PosAngle={-90,-10,190,150,180}](3,1){A}(2,3){B}(-1.5,2){C
    }(-3,-5){D}(-2.0,-2.2){E}
\pstGeneralConicEquation{A}{B}{C}{D}{E}{GCoefA,GCoefB,GCoefC,GCoefD,
    GCoefE,GCoefF}
\pstGeneralHyperbolaCoef[PosAngle=-30]{GCoefA,GCoefB,GCoefC,GCoefD,
    GCoefE,GCoefF}{O}{R}{RotAngle}
\pstGeneralHyperbola[linecolor=blue!40](O)(R)[RotAngle][72]
\pstGeneralConicTangentLine[PosAngle={-70}]{E}{GCoefA,GCoefB,GCoefC,
    GCoefD,GCoefE,GCoefF}{F}
\pstLineAB[linecolor=gray!60,nodesepA=-1]{E}{F}
\pstGeonode[PosAngle=-90](1.8,-1.5){T}
\pstGeneralConicTangentChord[PosAngle={20,60}]{T}{GCoefA,GCoefB,GCoefC
    ,GCoefD,GCoefE,GCoefF}{M}{N}
\pstLineAB[linecolor=red!60]{M}{N}
\pstLineAB[linecolor=gray!60,nodesepB=-1]{T}{M}
\pstLineAB[linecolor=gray!60,nodesepB=-1]{T}{N}
\end{pspicture}
```

4. Geometric Transformations

The geometric transformations are the ideal tools to construct geometric figures. All the classical transformations are available with the following macros which share the same syntaxic scheme end two parameters.

The common syntax put at the end two point lists whose second is optional or with a cardinal at least equal. These two lists contain the antecedent points and their respective images. In the case no image is given for some points the a default name is build appending a ' to the antecedent name.

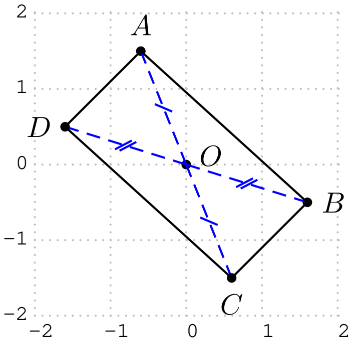
The first shared parameter is `CodeFig` which draws the specific constructions lines. Its default value is `false`, and a `true` value activates this optional drawing. The drawing is done using the line style `CodeFigStyle` (by default dashed), with the color `CodeFigColor` (by default cyan).

Their second shared parameter is `CurveType` which controls the drawing of a line crossing all images, and thus allow a quick description of a transformed figure.

4.1. Central symmetry

`\pstSymO [Options]` $\{O\}\{M_1, M_2, \cdots, M_n\}$ $[M_1', M_2', \cdots, M_p']$

Possible optional arguments are `PointSymbol`, `PosAngle`, `PointName`, `PointNameSep`, `PtNameMath`, `CodeFig`, `CodeFigColor`, and `CodeFigStyle`. Draw the symmetric point in relation to point O. The classical parameter of point creation are usable here, and also for all the following functions.

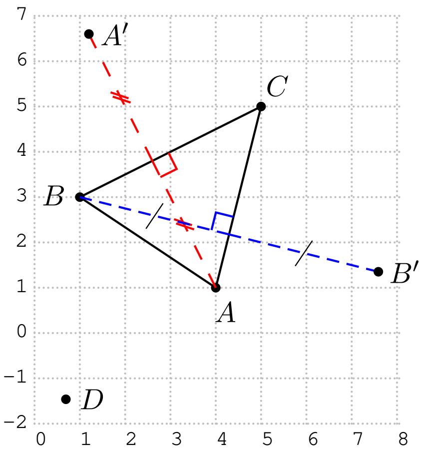



```
\begin{pspicture}[showgrid](-2,-2)(2,2)
\psset{CodeFig=true}
\pstGeonode[PosAngle={20,90,0}]{O}(-.6,1.5){A}(1.6,-.5){B}
\pstSymO[CodeFigColor=blue,
  PosAngle={-90,180}]{O}{A, B}[C, D]
\pstLineAB{A}{B}\pstLineAB{C}{D}
\pstLineAB{A}{D}\pstLineAB{C}{B}
\end{pspicture}
```

4.2. Orthogonal (or axial) symmetry

`\pstOrtSym [Options]` $\{A\}\{B\}\{M_1, M_2, \cdots, M_n\}$ $[M_1', M_2', \cdots, M_p']$

Possible optional arguments are `PointSymbol`, `PosAngle`, `PointName`, `PointNameSep`, `PtNameMath`, `CodeFig`, `CodeFigColor`, and `CodeFigStyle`. Draws the symmetric point in relation to line (AB).



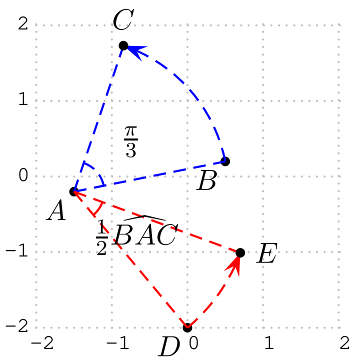

```
\psset{unit=0.6}
\begin{pspicture}[showgrid](0,-2)(8,7)
\pstTriangle(1,3){B}(5,5){C}(4,1){A}
\pstOrtSym{A}{B}{C}[D]
\psset{CodeFig=true}
\pstOrtSym[dash=2mm 2mm,CodeFigColor=red]%
  {C}{B}{A}
\pstOrtSym[SegmentSymbol=pstslash,
  linestyle=dotted,dotsep=3mm,CodeFigColor=blue]%
  {C}{A}{B}
\end{pspicture}
```

4.3. Rotation

\pstRotation [Options] $\{O\}\{M_1, M_2, \cdots, M_n\}$ $[M_1', M_2', \cdots, M_p']$
\pstAngleAOB$\{A\}\{O\}\{B\}$

Possible optional arguments are `PointSymbol`, `PosAngle`, `PointName`, `PointNameSep`, `PtNameMath`, and `RotAngle` for `\pstRotation` and `AngleCoef`, `RotAngle` for `\pstAngleABC`. Draw the image of M_i by the rotation of center O and angle given by the parameter `RotAngle`. This later can be an angle specified by three points. In such a case, the following function must be used:

Never forget to use the rotation for drawing a square or an equilateral triangle. The parameter `CodeFig` puts a bow with an arrow between the point and its image, and if `TransformLabel` (by default none) contain some text, it is put on the corresponding angle in mathematical mode.



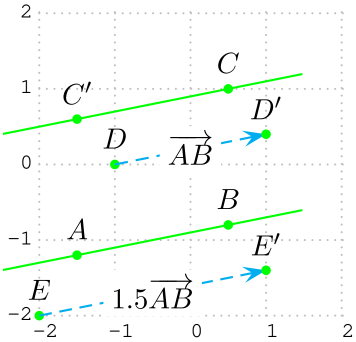

```
\begin{pspicture}[showgrid](-2,-2)(2,2)
\psset{arrowscale=2}
\pstGeonode[PosAngle=-135](-1.5,-.2){A}%
  (.5,.2){B}(0,-2){D}
\pstRotation[PosAngle=90,RotAngle=60,
  CodeFig,CodeFigColor=blue,
  TransformLabel=\frac{\pi}{3}]{A}{B}[C]
\pstRotation[AngleCoef=.5,
  RotAngle=\pstAngleAOB{B}{A}{C},
  CodeFigColor=red, CodeFig,
  TransformLabel=\frac{1}{2}\widehat{BAC}]{A}{D}[E]
\end{pspicture}
```

4.4. Translation

\pstTranslation [Options] $\{A\}\{B\}\{M_1, M_2, \cdots, M_n\}$ $[M_1', M_2', \cdots, M_p']$

Possible optional arguments are `PointSymbol`, `PosAngle`, `PointName`, `PointNameSep`, `PtNameMath`, and `DistCoef` Draws the translated M_i' of M_i using the vector $\vec{AB}$. Useful for drawing a parallel line.

The parameter `DistCoef` can be used as a multiplicand coefficient to modify the translation vector. The parameter `CodeFig` draws the translation vector le vecteur de translation between the point and its image, labeled in its middle defaultly with the vector name or by the text specified with `TransformLabel` (by default none).

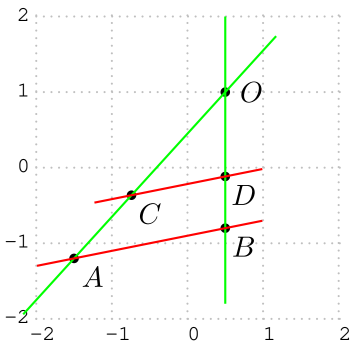



```
\begin{pspicture}[showgrid](-2,-2)(2,2)
\psset{linecolor=green,nodesep=-1,
  PosAngle=90,arrowscale=2}
\pstGeonode(-1.5,-1.2){A}(.5,-.8){B}(.5,1){C}(-1,0){D}(-2,-2){E}
\pstTranslation{B}{A}{C}
\psset{CodeFig,TransformLabel=default}
\pstTranslation{A}{B}{D}
\pstTranslation[DistCoef=1.5]{A}{B}{E}
\pstLineAB{A}{B}\pstLineAB{C}{C'}
\end{pspicture}
```

4.5. Homothetie

\pstHomO [Options] $\{O\}\{M_1, M_2, \cdots, M_n\}$ $[M'_1, M'_2, \cdots, M'_p]$

Possible optional arguments are `HomCoef`, `PointSymbol`, `PosAngle`, `PointName`, `PointNameSep`, `PtNameMath`, and `HomCoef`. Draws M'_i the image of M_i by the homotethy of center O and coefficient specified with the parameter `HomCoef`.

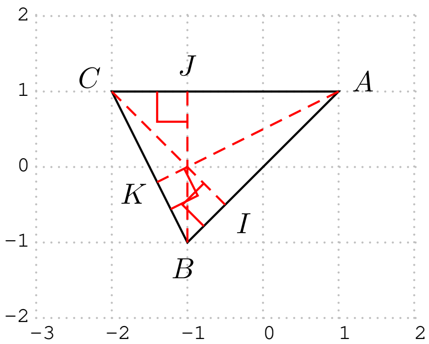



```
\begin{pspicture}[showgrid](-2,-2)(2,2)
\pstGeonode[PosAngle={0,-45}](.5,1){O}%
    (-1.5,-1.2){A}(.5,-.8){B}
\pstHomO[HomCoef=.62,PosAngle=-45]{O}{A,B}[C,D]
\psset{linecolor=green,nodesep=-1}
\pstLineAB{A}{O}\pstLineAB{B}{O}
\psset{linecolor=red,nodesep=-.5}
\pstLineAB{A}{B}\pstLineAB{C}{D}
\end{pspicture}
```

4.6. Orthogonal projection

\pstProjection [Options] $\{A\}\{B\}\{M_1, M_2, \cdots, M_n\}$ $[M'_1, M'_2, \cdots, M'_p]$

Possible optional arguments are `PointSymbol`, `PosAngle`, `PointName`, `PointNameSep`, `PtNameMath`, `CodeFig`, `CodeFigColor`, and`CodeFigStyle` Projects orthogonally the point M_i on the line (AB). Useful for the altitude of a triangle. The name is aligned with the point and the projected point as shown in the exemple.



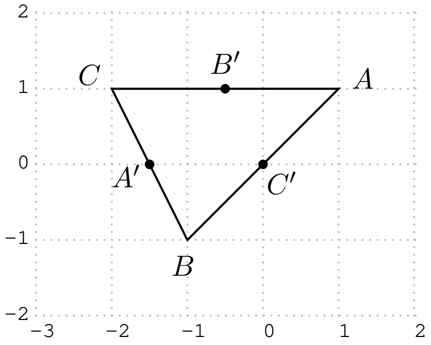

```
\begin{pspicture}[showgrid](-3,-2)(2,2)
\psset{PointSymbol=none,CodeFig,CodeFigColor=red}
\pstTriangle(1,1){A}(-2,1){C}(-1,-1){B}
\pstProjection{A}{B}{C}[I]
\pstProjection{A}{C}{B}[J]
\pstProjection{C}{B}{A}[K]
\end{pspicture}
```

5. Special object

5.1. Midpoint

\pstMiddleAB [Options] $\{A\}\{B\}\{I\}$

`PointSymbol`, `PosAngle`, `PointName`, `PointNameSep`, `PtNameMath`, `SegmentSymbol`, `CodeFig`, `CodeFigColor`, and `CodeFigStyle` Draw the midpoint I of segment $[AB]$. By default, the point name is automatically put below the segment.

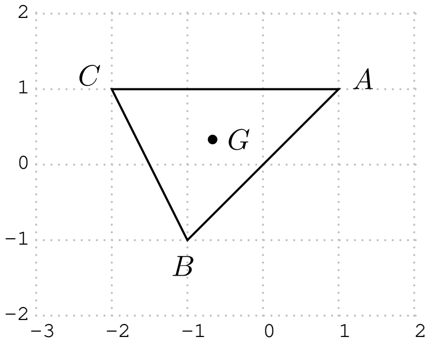



```
\begin{pspicture}[showgrid](-3,-2)(2,2)
\pstTriangle[PointSymbol=none]%
  (1,1){A}(-1,-1){B}(-2,1){C}
\pstMiddleAB{A}{B}{C'}
\pstMiddleAB{C}{A}{B'}
\pstMiddleAB{B}{C}{A'}
\end{pspicture}
```

5.2. Triangle center of gravity

\pstCGravABC [Options] $\{A\}\{B\}\{C\}\{G\}$

Possible optional arguments are PointName, PointNameSep, PosAngle, PointSymbol, and PtNameMath Draw the ABC triangle centre of gravity G.

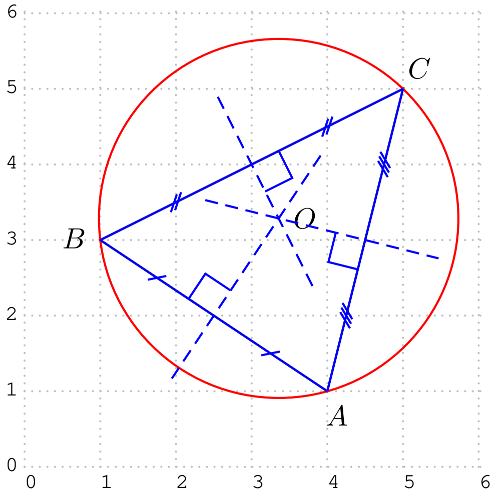



```
\begin{pspicture}[showgrid](-3,-2)(2,2)
\pstTriangle[PointSymbol=none]%
  (1,1){A}(-1,-1){B}(-2,1){C}
\pstCGravABC{A}{B}{C}{G}
\end{pspicture}
```

5.3. Centre of the circumcircle of a triangle

\pstCircleABC [Options] $\{A\}\{B\}\{C\}\{O\}$

Possible optional arguments are PointName, PointNameSep, PosAngle, PointSymbol, PtNameMath, DrawCirABC, CodeFig, CodeFigColor, CodeFigStyle, SegmentSymbolA, SegmentSymbolB, and SegmentSymbolC. Draws the circle crossing three points (the circum circle) and put its center O. The effective drawing is controlled by the boolean parameter DrawCirABC (by default true). Moreover the intermediate constructs (mediator lines) can be drawn by setting the boolean parameter CodeFig. In that case the middle points are marked on the segemnts using three different marks given by the parameters SegmentSymbolA, SegmentSymbolB et SegmentSymbolC.



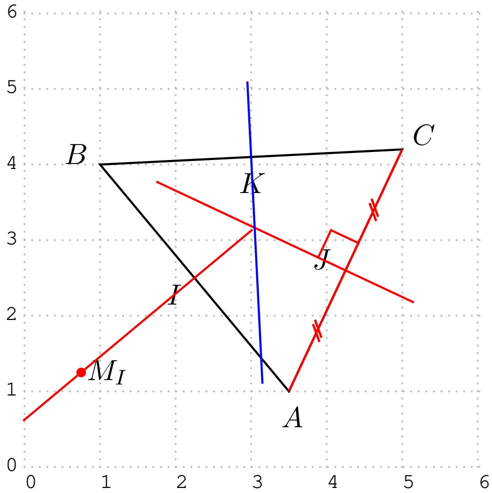

```
\begin{pspicture}[showgrid](6,6)
\pstTriangle[PointSymbol=none]%
  (4,1){A}(1,3){B}(5,5){C}
\pstCircleABC[CodeFig,CodeFigColor=blue,
  linecolor=red,PointSymbol=none]{A}{B}{C}{O}
\end{pspicture}
```

5.4. Perpendicular bisector of a segment

\pstMediatorAB [Options] $\{A\}\{B\}\{I\}\{M\}$

Possible optional arguments are PointName, PointNameSep, PosAngle, PointSymbol, PtNameMath, CodeFig, CodeFigColor, CodeFigStyle, and SegmentSymbol. The perpendicular bisector of a segment is a line perpendicular to this segment in its midpoint. The segment is $[AB]$, the midpoint I, and M is a point belonging to the perpendicular bisector line. It is build by a rotation of B of 90 degrees around I. This mean that the order of A and B is important, it controls the position of M. The command creates the two points M end I. The construction is controlled by the following parameters:

- CodeFig, CodeFigColor and SegmentSymbol for marking the right angle ;
- PointSymbol et PointName for controlling the drawing of the two points, each of them can be specified separately with the parameters ...A and ...B ;
- parameters controlling the line drawing.

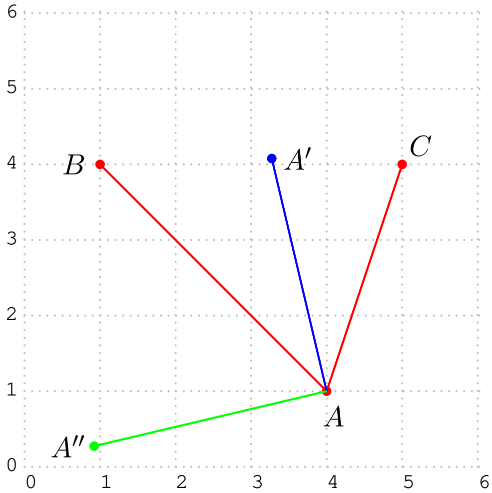



```
\begin{pspicture}[showgrid](6,6)
\pstTriangle[PointSymbol=none](3.5,1){A}(1,4){B}(5,4.2){C}
\psset{linecolor=red,CodeFigColor=red,nodesep=-1}
\pstMediatorAB[PointSymbolA=none]{A}{B}{I}{M_I}
\psset{PointSymbol=none,PointNameB=none}
\pstMediatorAB[CodeFig=true]{A}{C}{J}{M_J}
\pstMediatorAB[PosAngleA=45,linecolor=blue]
      {C}{B}{K}{M_K}
\end{pspicture}
```

5.5. Bisectors of angles

\pstBissectBAC [Options] $\{B\}\{A\}\{C\}\{N\}$
\pstOutBissectBAC [Options] $\{B\}\{A\}\{C\}\{N\}$

Possible optional arguments are PointSymbol, PosAngle, PointName, PointNameSep, and PtNameMath. There are two bisectors for a given geometric angle: the inside one and the outside one; this is why there is two commands. The angle is specified by three points specified in the trigonometric direction (anti-clockwise). The result of the commands is the specific line and a point belonging to this line. This point is built by a rotation of point B.




```
\begin{pspicture}[showgrid](6,6)
\psset{CurveType=polyline,linecolor=red}
\pstGeonode[PosAngle={180,-75,45}]%
  (1,4){B}(4,1){A}(5,4){C}
\pstBissectBAC[linecolor=blue]{C}{A}{B}{A'}
\pstOutBissectBAC[linecolor=green,PosAngle=180]%
  {C}{A}{B}{A''}
\end{pspicture}
```

6. Intersections

Points can be defined by intersections. Six intersection types are managed:

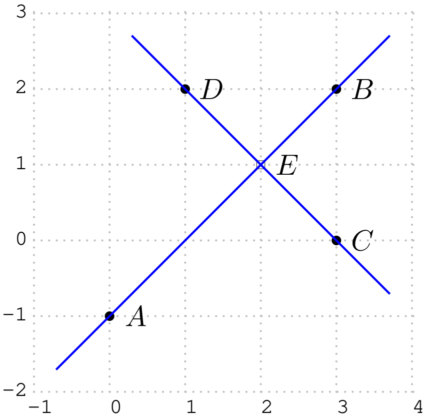
- line-line;
- line-circle;
- circle-circle;
- function-function;
- function-line;
- function-circle.

An intersection can not exist: case of parallel lines. In such a case, the point(s) are positioned at the origin. In fact, the user has to manage the existence of these points.

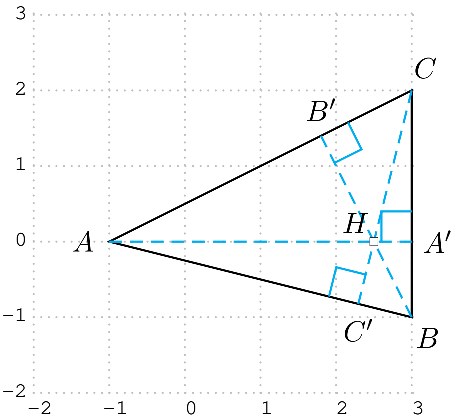
6.1. Line-Line

`\pstInterLL [Options]` $\{A\}\{B\}\{C\}\{D\}\{M\}$

Possible optional arguments are `PointSymbol`, `PosAngle`, `PointName`, `PointNameSep`, and `PtNameMath`. Draw the intersection point between lines (AB) and (CD).




```
\begin{pspicture}[showgrid](-1,-2)(4,3)
\pstGeonode(0,-1){A}(3,2){B}(3,0){C}(1,2){D}
\pstInterLL[PointSymbol=square]{A}{B}{C}{D}{E}
\psset{linecolor=blue, nodesep=-1}
\pstLineAB{A}{B}\pstLineAB{C}{D}
\end{pspicture}
```




```
\begin{pspicture}[showgrid](-2,-2)(3,3)
\psset{CodeFig,PointSymbol=none}
\pstTriangle[PosAngleA=180](-1,0){A}(3,-1){B}(3,2){C}
\pstProjection[PosAngle=-90]{B}{A}{C}
\pstProjection{B}{C}{A}
\pstProjection[PosAngle=90]{A}{C}{B}
\pstInterLL[PosAngle=135,PointSymbol=square]{A}{A'}{B}{B'}{
    H}
\end{pspicture}
```

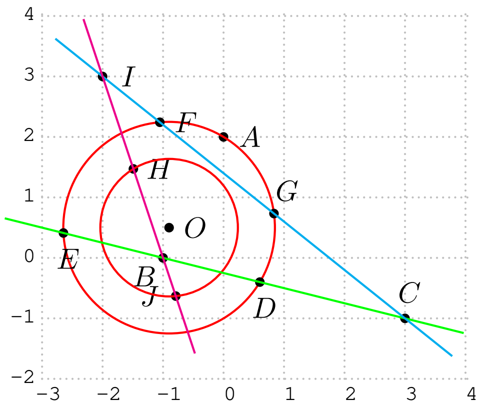
6.2. Circle–Line

\pstInterLC [Options] $\{A\}\{B\}\{O\}\{C\}\{M_1\}\{M_2\}$

Possible optional arguments are PointSymbol, PosAngle, PointName, PointNameSep, PtNameMath, PointSymbolA, PosAngleA, PointNameA, PointSymbolB, PosAngleB, PointNameB, Radius, and Diameter. Draw the one or two intersection point(s) between the line (AB) and the circle of centre O and with radius OC.

The circle is specified with its center and either a point of its circumference or with a radius specified with parameter radius or its diameter specified with parameter Diameter. These two parameters can be specified by macros \pstDist,\pstDistMul,\pstDistAdd, \pstDistSub etc.

The position of the wo points is such that the vectors $\vec{AB}$ abd $M_1\vec{M}_2$ are in the same direction. Thus, if the points definig the line are switch, then the resulting points will be also switched. If the intersection is void, then the points are positionned at the center of the circle.

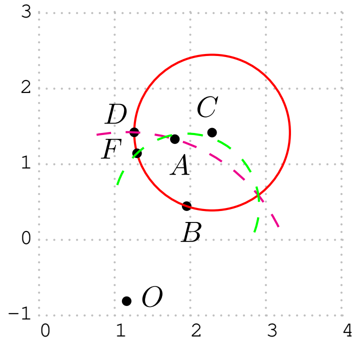



```
\psset{unit=0.8}
\begin{pspicture}[showgrid](-3,-2)(4,4)
\pstGeonode[PosAngle={-135,80,0}](-1,0){B}(3,-1){C}(-.9,.5){O
    }(0,2){A}
\pstGeonode(-2,3){I}
\pstCircleOA[linecolor=red]{O}{A}
\pstInterLC[PosAngle=-80]{C}{B}{O}{A}{D}{E}
\pstInterLC[PosAngleB=60, Radius=\pstDistAB{O}{D}]{I}{C}{O}{}{F}{
    G}
\pstInterLC[PosAngleB=180,Diameter=\pstDistMul{O}{D}{1.3}]
  {I}{B}{O}{}{H}{J}
\pstCircleOA[linecolor=red,Diameter=\pstDistMul{O}{D}{1.3}]{O}{}
\psset{nodesep=-1}
\pstLineAB[linecolor=green]{E}{C}
\pstLineAB[linecolor=cyan]{I}{C}
\pstLineAB[linecolor=magenta]{J}{I}
\end{pspicture}
```

6.3. Circle–Circle

\pstInterCC [Options] $\{O_1\}\{B\}\{O_2\}\{C\}\{M_1\}\{M_2\}$

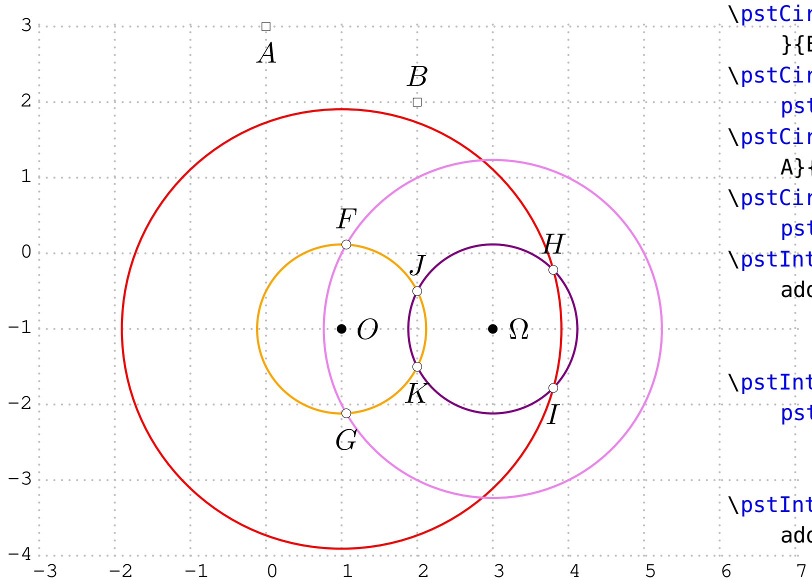
This function is similar to the last one. The boolean parameters CodeFigA et CodeFigB allow the drawing of the arcs at the intersection. In order to get a coherence CodeFig allow the drawing of both arcs. The boolean parameters CodeFigAarc and CodeFigBarc specified the direction of these optional arcs: trigonometric (by default) or clockwise. Here is a first example.




```
\begin{pspicture}[showgrid](0,-1)(4,3)
\psset{dash=2mm 2mm}
\rput{10}{%
  \pstGeonode[PosAngle={0,-90,-90,90}]
    (1,-1){O}(2,1){A}(2,0.1){B}(2.5,1){C}}
\pstCircleOA[linecolor=red]{C}{B}
\pstInterCC[PosAngleA=135, CodeFigA=true, CodeFigAarc=false,
  CodeFigColor=magenta]{O}{A}{C}{B}{D}{E}
\pstInterCC[PosAngleA=170, CodeFigA=true,
   CodeFigAarc=false,
   CodeFigColor=green]{B}{E}{C}{B}{F}{G}
\end{pspicture}
```

And a more complete one, which includes the special circle specification using radius and diameter. For such specifications it exists the parameters `RadiusA`, `RadiusB`, `DiameterA` and `DiameterB`.

The macro `\pstInterCC` will not display the intersections as default, if you want to display the label or symbol of the intersections, you must setup the parameters `PosAngleA` and `PosAngleB` to change the default behavior.



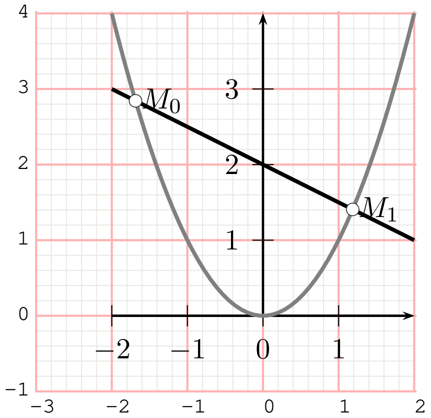

```
\begin{pspicture}[showgrid](-3,-4)(7,3)
\pstGeonode[PointName={\Omega,O}](3,-1){Omega
    }(1,-1){O}
\pstGeonode[PointSymbol=square, PosAngle
    ={-90,90}](0,3){A}(2,2){B}
\psset{PointSymbol=o}
\pstCircleOA[linecolor=red, Radius=\pstDistMul{A
    }{B}{1 3 10 div add}]{O}{}
\pstCircleOA[linecolor=Orange, Diameter=\
    pstDistAB{A}{B}]{O}{}
\pstCircleOA[linecolor=Violet, Radius=\pstDistAB{
    A}{B}]{Omega}{}
\pstCircleOA[linecolor=Purple, Diameter=\
    pstDistAB{A}{B}]{Omega}{}
\pstInterCC[RadiusA=\pstDistMul{A}{B}{1 3 10 div
    add},
         RadiusB=\pstDistAB{A}{B}]{O}{}{Omega}{}{
            D}{E}
\pstInterCC[DiameterA=\pstDistAB{A}{B}, RadiusB=\
    pstDistAB{A}{B},
         PosAngleA=90,PosAngleB=-90]{O}{}{Omega
            }{}{F}{G}
\pstInterCC[RadiusA=\pstDistMul{A}{B}{1 3 10 div
    add}, DiameterB=\pstDistAB{A}{B},
         PosAngleA=90,PosAngleB=-90]{O}{}{Omega
            }{}{H}{I}
\pstInterCC[DiameterA=\pstDistAB{A}{B}, DiameterB
    =\pstDistAB{A}{B},
         PosAngleA=90,PosAngleB=-90]{O}{}{Omega
            }{}{J}{K}
\end{pspicture}
```

6.4. Function–function

`\pstInterFF [Options]` $\{f\}\{g\}\{x_0\}\{M\}$

This function put a point at the intersection between two curves defined by a function. x_0 is an intersection approximated value of the abscissa. It is obviously possible to use this function several time if more than one intersection is present. Each function is describerd in `PostScript`in the same way as the description used by the `\psplot` macro of `PSTricks`. A constant function can be specified, and then seaching function root is possible.

The Newton algorithm is used for the research, and the intersection may not to be found. In such a case the point is positionned at the origin. On the other hand, the research can be trapped (in a local extremum near zero).

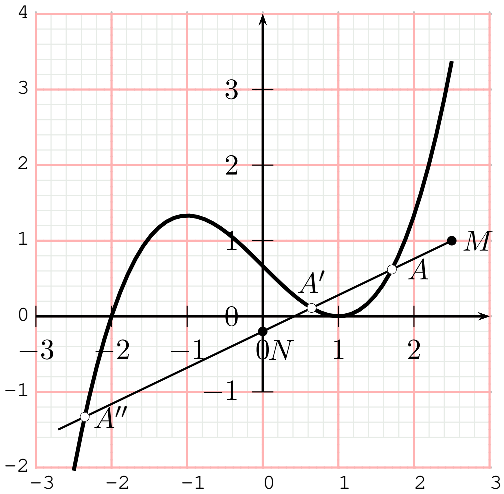



```
\begin{pspicture}[showgrid](-3,-1)(2,4)
\psaxes{->}(0,0)(-2,0)(2,4)
\psset{linewidth=1.5pt,algebraic}
\psplot[linecolor=gray]{-2}{2}{x^2}
\psplot{-2}{2}{2-x/2}
\psset{PointSymbol=o}
\pstInterFF{2-x/2}{x^2}{1}{M_1}
\pstInterFF{2-x/2}{x^2}{-2}{M_0}
\end{pspicture}
```

6.5. Function–line

`\pstInterFL [Options] `$\{f\}\{A\}\{B\}\{x_0\}\{M\}$

Puts a point at the intersection between the function f and the line (AB).

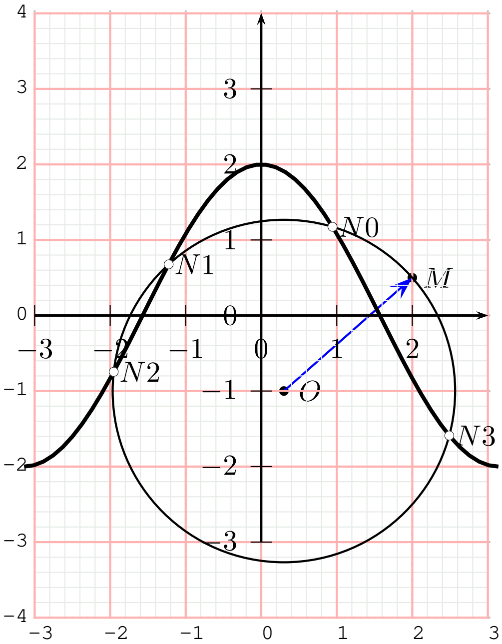



```
\begin{pspicture}[showgrid](-3,-1.5)(3,4)
\def\F{x^3/3 - x + 2/3 }
\psaxes{->}(0,0)(-3,-1)(3,4)
\psplot[linewidth=1.5pt,algebraic]{-2.5}{2.5}{\F}
\psset{PointSymbol=*}
\pstGeonode[PosAngle={-45,0}](0,-.2){N}(2.5,1){M}
\pstLineAB[nodesepA=-3cm]{N}{M}
\psset{PointSymbol=o,algebraic}
\pstInterFL{\F}{N}{M}{2}{A}
\pstInterFL[PosAngle=90]{\F}{N}{M}{0}{A'}
\pstInterFL{\F}{N}{M}{-2}{A''}
\end{pspicture}
```

6.6. Function–Circle

`\pstInterFC [Options] `$\{f\}\{O\}\{A\}\{x_0\}\{M\}$

Puts a point at the intersection between the function f and the circle of centre O and radius OA.

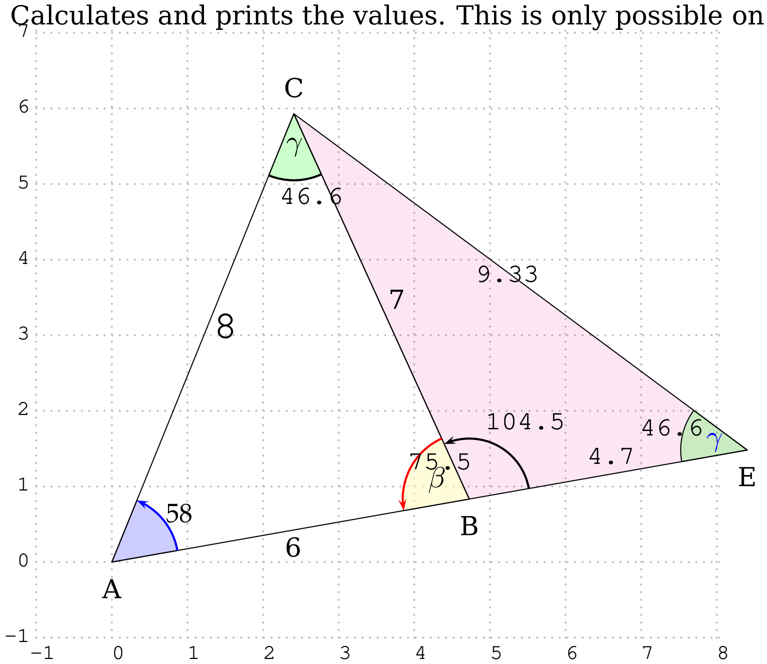



```
\begin{pspicture}[showgrid](-3,-4)(3,4)
\def\F{2*cos(x)}
\psset{algebraic}
\pstGeonode(0.3,-1){O}(2,.5){M}
\ncline[linecolor=blue, arrowscale=2]{->}{O}{M}
\psaxes{->}(0,0)(-3,-3)(3,4)
\psplot[linewidth=1.5pt]{-3.14}{3.14}{\F}
\pstCircleOA[PointSymbol=*]{O}{M}
\psset{PointSymbol=o}
\pstInterFC{\F}{O}{M}{1}{N0}
\pstInterFC{\F}{O}{M}{-1}{N1}
\pstInterFC{\F}{O}{M}{-2}{N2}
\pstInterFC{\F}{O}{M}{2}{N3}
\end{pspicture}
```

7. Helper Macros

\psGetDistanceAB [Options] $(x_1,y_1)(x_2,y_2)$\{<*name*>\}
\psGetAngleABC [Options] $(x_1,y_1)(x_2,y_2)(x_3,y_3)$\{<*symbol*>\}

Calculates and prints the values. This is only possible on PostScript level!



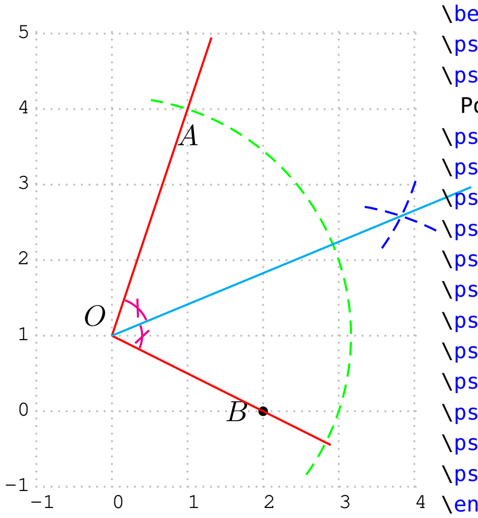

```
\begin{pspicture}(-1,0)(11,8)
\psgrid[gridlabels=0pt,subgriddiv=2,gridwidth=0.4pt,subgridwidth=0.2pt,gridcolor=black!60,
    subgridcolor=black!40]
\def\sideC{6} \def\sideA{7} \def\sideB{8}
\psset{PointSymbol=none,linejoin=1,linewidth=0.4pt,PtNameMath=false,labelsep=0.07,MarkAngleRadius
    =1.1,decimals=1,comma}
\pstGeonode[PosAngle={-90,-90}](0,0){A}(\sideC;10){B}
\pstInterCC[RadiusA=\pstDistVal{\sideB},RadiusB=\pstDistVal{\sideA},PosAngle=90,PointNameA=C]{A}{}{B
    }{}{C}{C-}
\pstInterCC[RadiusA=\pstDistAB{A}{B},RadiusB=\pstDistAB{B}{C}]{C}{}{A}{}{D-}{D}
\pstInterLC[Radius=\pstDistAB{A}{C}]{C}{D}{C}{}{A'-}{A'}
```

```
\pstInterCC[RadiusA=\pstDistAB{A}{B},RadiusB=\pstDistAB{B}{C}]{A'}{}{C}{}{B'}{B'-}
\pstInterLL[PosAngle=90,PointName=default]{B'}{C}{A}{B}{E}
\pspolygon(A)(B)(C)
\pspolygon[fillstyle=solid,fillcolor=magenta,opacity=0.1](C)(E)(B)
%
\psGetAngleABC[ArcColor=blue,AngleValue=true,LabelSep=0.8,arrows=->,decimals=0,PSfont=Palatino-Roman
    ](B)(A)(C){}
\psGetAngleABC[AngleValue=true,ArcColor=red,arrows=->,WedgeOpacity=0.6,WedgeColor=yellow!30,LabelSep
    =0.5](C)(B)(A){$\beta$}
\psGetAngleABC[LabelSep=0.8,WedgeColor=green,xShift=-6,yShift=-10](A)(C)(B){$\gamma$}
\psGetAngleABC[LabelSep=0.8,AngleArc=false,WedgeColor=green,arrows=->,xShift=-15,yShift=0](C)(E)(B)
    {\color{blue}$\gamma$}
\psGetAngleABC[AngleValue=true,MarkAngleRadius=1.0,LabelSep=0.5,ShowWedge=false,xShift=-5,yShift=7,
    arrows=->](E)(B)(C){}
%
\pcline[linestyle=none](A)(B)\nbput{\sideC}
\pcline[linestyle=none](C)(B)\naput{\sideA}
\psGetDistanceAB[xShift=-8,yShift=4](B)(E){MW}
\psGetDistanceAB[fontscale=15,xShift=4,decimals=0](A)(C){MAC}
\psGetDistanceAB[xShift=-17,decimals=2](E)(C){MEC}
\end{pspicture}
```

Part II.
Examples gallery

A. Basic geometry

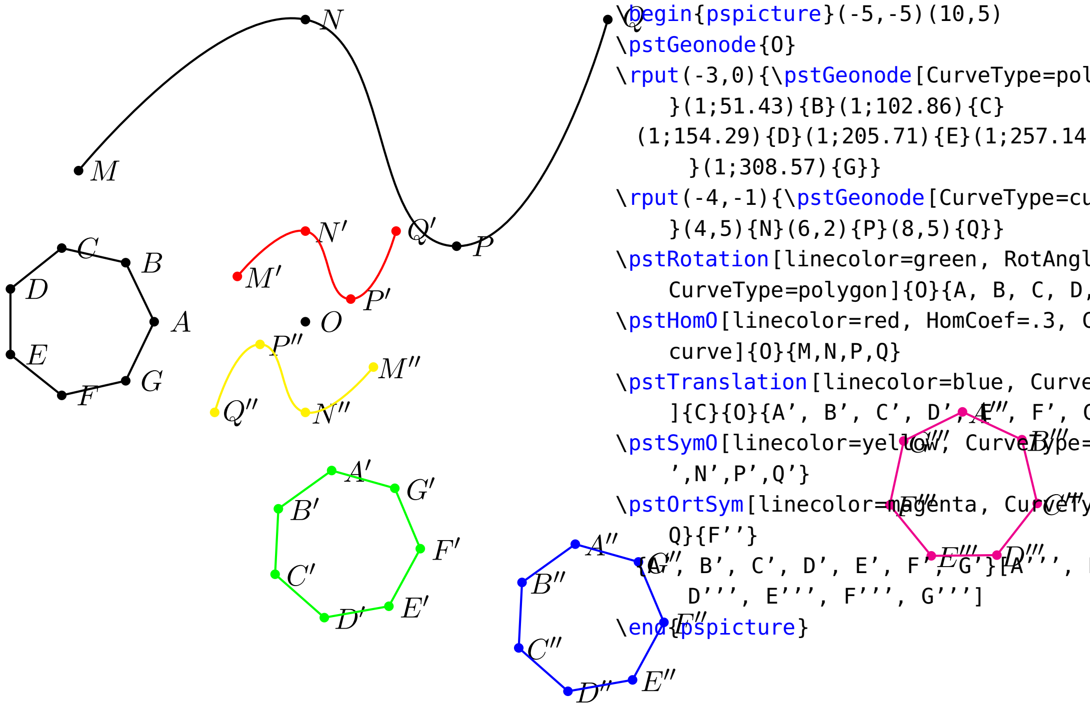
A.1. Drawing of the bissector




```
\begin{pspicture}[showgrid](-1,-1)(4.4,5)
\psset{PointSymbol=none,PointName=none}
\pstGeonode[PosAngle={180,130,-90},PointSymbol={*,none},
  PointName=default](2,0){B}(0,1){O}(1,4){A}
\pstLineAB[nodesepB=-1,linecolor=red]{O}{A}
\pstLineAB[nodesepB=-1,linecolor=red]{O}{B}
\pstInterLC[PosAngleB=-45]{O}{B}{O}{A}{G}{C}
\psset{arcsepA=-1, arcsepB=-1}
\pstArcOAB[linecolor=green,linestyle=dashed]{O}{C}{A}
\pstInterCC[PosAngleA=100]{A}{O}{C}{O}{O'}{OO}
\pstArcOAB[linecolor=blue,linestyle=dashed]{A}{O'}{O'}
\pstArcOAB[linecolor=blue,linestyle=dashed]{C}{O'}{O'}
\pstLineAB[nodesepB=-1,linecolor=cyan]{O}{O'}
\psset{arcsep=1pt,linecolor=magenta,Mark=MarkHash}
\pstMarkAngle{C}{O}{O'}{}
\pstMarkAngle[MarkAngleRadius=.5]{O'}{O}{A}{}
\end{pspicture}
```

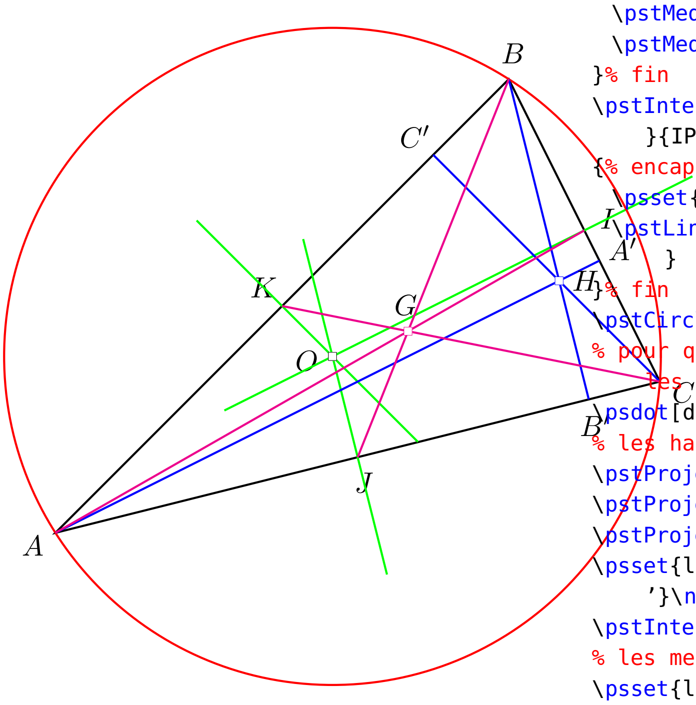
A.2. Transformation de polygones et courbes

Here is an example of the use of `CurveType` with transformation.



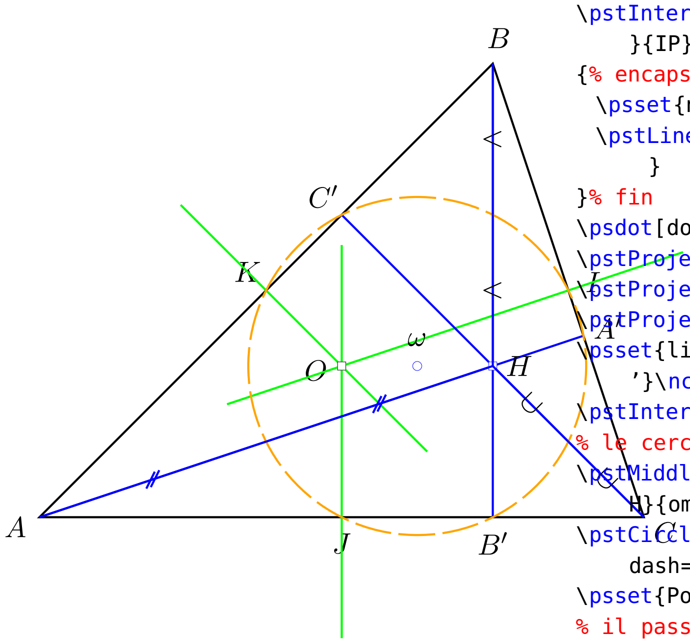

```
\begin{pspicture}(-5,-5)(10,5)
\pstGeonode{O}
\rput(-3,0){\pstGeonode[CurveType=polygon](1,0){A
    }(1;51.43){B}(1;102.86){C}
  (1;154.29){D}(1;205.71){E}(1;257.14){F
      }(1;308.57){G}}
\rput(-4,-1){\pstGeonode[CurveType=curve](1,3){M
    }(4,5){N}(6,2){P}(8,5){Q}}
\pstRotation[linecolor=green, RotAngle=100,
    CurveType=polygon]{O}{A, B, C, D, E, F, G}
\pstHomO[linecolor=red, HomCoef=.3, CurveType=
    curve]{O}{M,N,P,Q}
\pstTranslation[linecolor=blue, CurveType=polygon
    ]{C}{O}{A', B', C', D', E', F', G'}
\pstSymO[linecolor=yellow, CurveType=curve]{O}{M
    ',N',P',Q'}
\pstOrtSym[linecolor=magenta, CurveType=polygon]{
    Q}{F''}
  {A', B', C', D', E', F', G'}[A''', B''', C''',
      D''', E''', F''', G''']
\end{pspicture}
```

A.3. Triangle lines




```
\psset{unit=2}
\begin{pspicture}(-3,-2)(3,3)
\psset{PointSymbol=none}
\pstTriangle[PointSymbol=none](-2,-1){A}(1,2){B
    }(2,0){C}
{ \psset{linestyle=none, PointNameB=none}
  \pstMediatorAB{A}{B}{K}{KP}
  \pstMediatorAB[PosAngleA=-40]{C}{A}{J}{JP}
  \pstMediatorAB[PosAngleA=75]{B}{C}{I}{IP}
}% fin
\pstInterLL[PointSymbol=square, PosAngle=-170]{I
    }{IP}{J}{JP}{O}
{% encapsulation de modif parametres
  \psset{nodesep=-.8, linecolor=green}
  \pstLineAB{O}{I}\pstLineAB{O}{J}\pstLineAB{O}{K
    }
}% fin
\pstCircleOA[linecolor=red]{O}{A}
% pour que le symbol de O soit sur et non sous
    les droites
\psdot[dotstyle=square](O)
% les hauteurs et l'orthocentre
\pstProjection{B}{A}{C}
\pstProjection{B}{C}{A}
\pstProjection{A}{C}{B}
\psset{linecolor=blue}\ncline{A}{A'}\ncline{C}{C
    '}\ncline{B}{B'}
\pstInterLL[PointSymbol=square]{A}{A'}{B}{B'}{H}
% les medianes et le centre de gravite
\psset{linecolor=magenta}\ncline{A}{I}\ncline{C}{
    K}\ncline{B}{J}
\pstCGravABC[PointSymbol=square, PosAngle=95]{A}{
    B}{C}{G}
\end{pspicture}
```

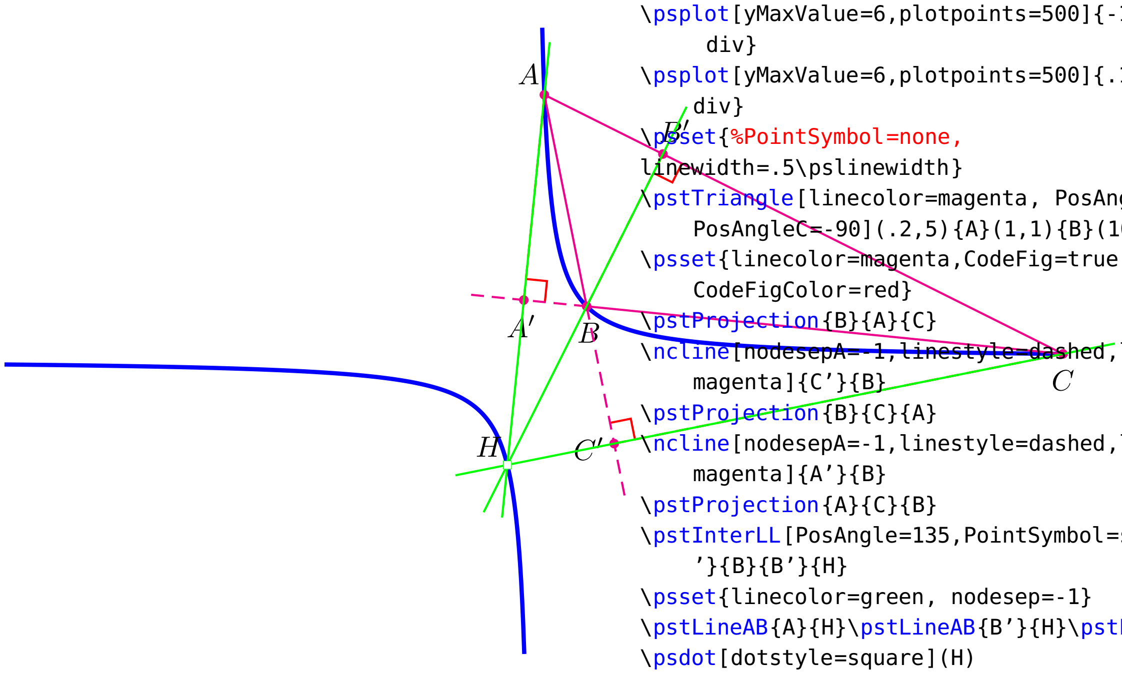
A.4. Euler circle




```
\psset{unit=2}
\begin{pspicture}(-3,-1.5)(3,2.5)
\psset{PointSymbol=none}
\pstTriangle(-2,-1){A}(1,2){B}(2,-1){C}
{% encapsulation de modif parametres
  \psset{linestyle=none, PointSymbolB=none,
      PointNameB=none}
  \pstMediatorAB{A}{B}{K}{KP}
  \pstMediatorAB{C}{A}{J}{JP}
  \pstMediatorAB{B}{C}{I}{IP}
}% fin
\pstInterLL[PointSymbol=square, PosAngle=-170]{I
    }{IP}{J}{JP}{O}
{% encapsulation de modif parametres
  \psset{nodesep=-.8, linecolor=green}
  \pstLineAB{O}{I}\pstLineAB{O}{J}\pstLineAB{O}{K
      }
}% fin
\psdot[dotstyle=square](O)
\pstProjection{B}{A}{C}
\pstProjection{B}{C}{A}
\pstProjection{A}{C}{B}
\psset{linecolor=blue}\ncline{A}{A'}\ncline{C}{C
    '}\ncline{B}{B'}
\pstInterLL[PointSymbol=square]{A}{A'}{B}{B'}{H}
% le cercle d'Euler (centre au milieu de [OH])
\pstMiddleAB[PointSymbol=o, PointName=\omega]{O}{
    H}{omega}
\pstCircleOA[linecolor=Orange, linestyle=dashed,
    dash=5mm 1mm]{omega}{B'}
\psset{PointName=none}
% il passe par le milieu des segments joignant l'
    orthocentre et les sommets
\pstMiddleAB{H}{A}{AH}\pstMiddleAB{H}{B}{BH}\
    pstMiddleAB{H}{C}{CH}
\pstSegmentMark{H}{AH}\pstSegmentMark{AH}{A}
\psset{SegmentSymbol=wedge}\pstSegmentMark{H}{BH
    }\pstSegmentMark{BH}{B}
\psset{SegmentSymbol=cup}\pstSegmentMark{H}{CH}\
    pstSegmentMark{CH}{C}
\end{pspicture}
```

A.5. Orthocenter and hyperbola

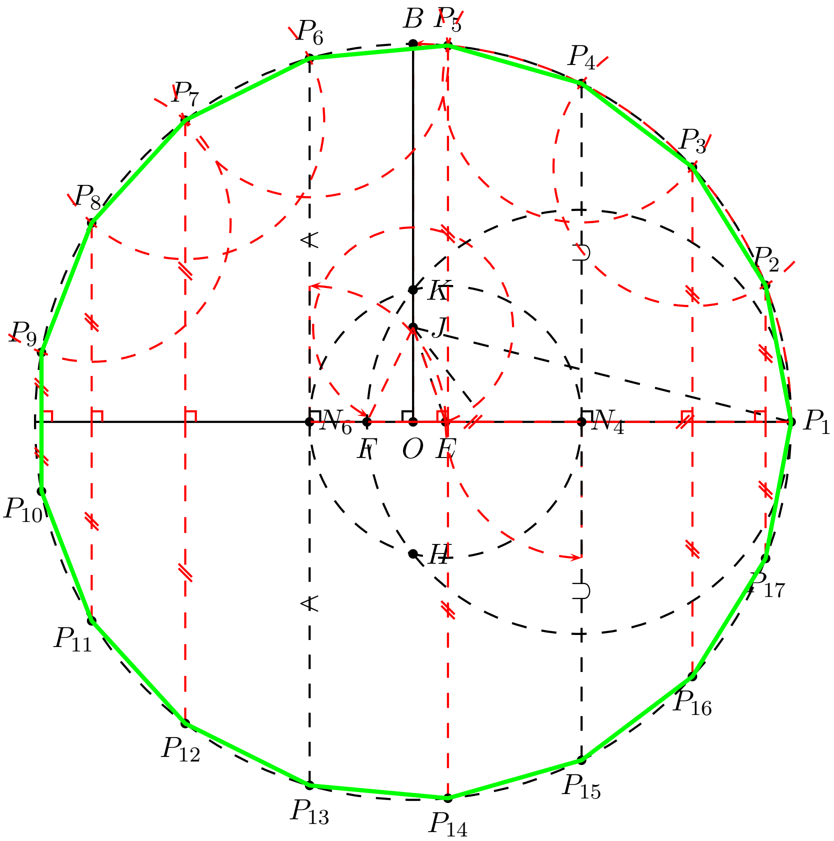
The orthocenter of a triangle whose points are on the branches of the hyperbola $\mathcal{H} : y = a/x$ belong to this hyperbola.




```
\psset{unit=0.7}
\begin{pspicture}(-11,-5)(11,7)
\psset{linecolor=blue, linewidth=2\pslinewidth}
\psplot[yMaxValue=6,plotpoints=500]{-10}{-.1}{1 x div}
\psplot[yMaxValue=6,plotpoints=500]{.1}{10}{1 x div}
\psset{%PointSymbol=none,
linewidth=.5\pslinewidth}
\pstTriangle[linecolor=magenta, PosAngleB=-85,
    PosAngleC=-90](.2,5){A}(1,1){B}(10,.1){C}
\psset{linecolor=magenta,CodeFig=true,
    CodeFigColor=red}
\pstProjection{B}{A}{C}
\ncline[nodesepA=-1,linestyle=dashed,linecolor=
    magenta]{C'}{B}
\pstProjection{B}{C}{A}
\ncline[nodesepA=-1,linestyle=dashed,linecolor=
    magenta]{A'}{B}
\pstProjection{A}{C}{B}
\pstInterLL[PosAngle=135,PointSymbol=square]{A}{A'}{B}{B'}{H}
\psset{linecolor=green, nodesep=-1}
\pstLineAB{A}{H}\pstLineAB{B'}{H}\pstLineAB{C}{H}
\psdot[dotstyle=square](H)
\end{pspicture}
```

A.6. 17 sides regular polygon

Striking picture created by K. F. Gauss. he also prooved that it is possible to build the regular polygons which have $2^{2^p}+1$ sides, the following one has 257 sides!

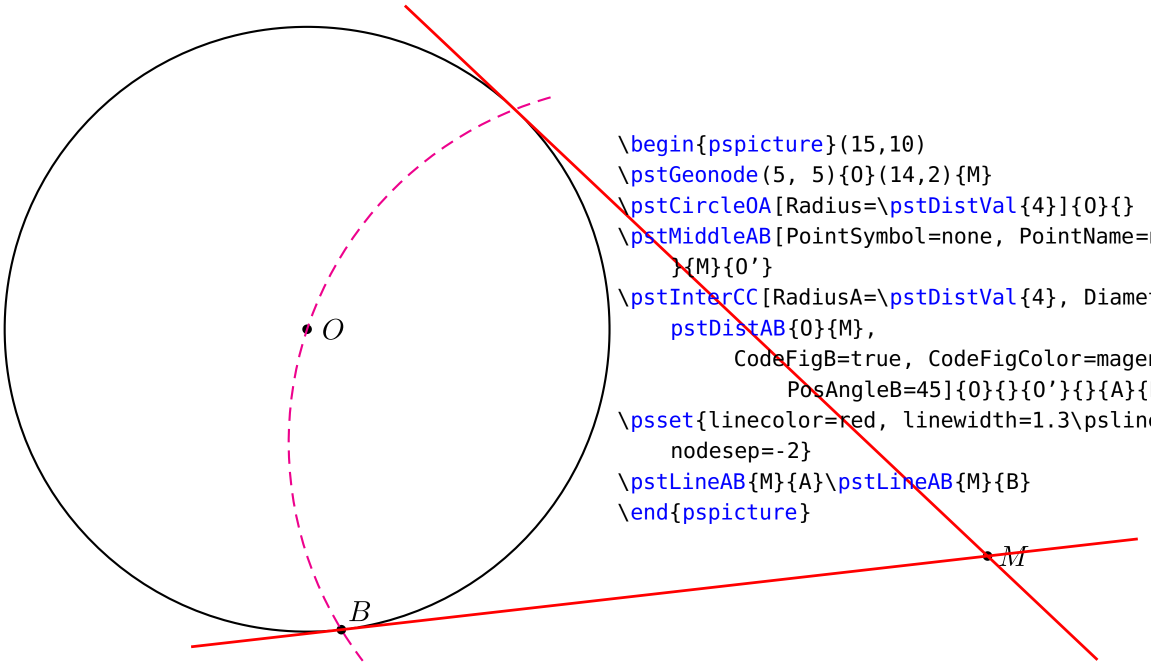



```
\begin{pspicture}(-5.5,-5.5)(5.5,6)
  \psset{CodeFig, RightAngleSize=.14, CodeFigColor=red,
    CodeFigB=true, linestyle=dashed, dash=2mm 2mm}
  \pstGeonode[PosAngle={-90,0}]{O}(5;0){P_1}
  \pstCircleOA{O}{P_1}
  \pstSymO[PointSymbol=none, PointName=none, CodeFig=false]{O}{P_1}[PP_1]
  \ncline[linestyle=solid]{PP_1}{P_1}
  \pstRotation[RotAngle=90, PosAngle=90]{O}{P_1}[B]
  \pstRightAngle[linestyle=solid]{B}{O}{PP_1}\ncline[linestyle=solid]{O}{B}
  \pstHomO[HomCoef=.25]{O}{B}[J] \ncline{J}{P_1}
  \pstBissectBAC[PointSymbol=none, PointName=none]{O}{J}{P_1}{PE1}
  \pstBissectBAC[PointSymbol=none, PointName=none]{O}{J}{PE1}{PE2}
  \pstInterLL[PosAngle=-90]{O}{P_1}{J}{PE2}{E}
  \pstRotation[PosAngle=-90, RotAngle=-45, PointSymbol=none, PointName=none]{J}{E}[PF1]
  \pstInterLL[PosAngle=-90]{O}{P_1}{J}{PF1}{F}
  \pstMiddleAB[PointSymbol=none, PointName=none]{F}{P_1}{MFP1} \pstCircleOA{MFP1}{P_1}
  \pstInterLC[PointSymbolA=none, PointNameA=none]{O}{B}{MFP1}{P_1}{H}{K}
  \pstCircleOA{E}{K} \pstInterLC{O}{P_1}{E}{K}{N_6}{N_4}
  \pstRotation[RotAngle=90,PointSymbol=none, PointName=none]{N_6}{E}[PP_6]
  \pstInterLC[PosAngleA=90,PosAngleB=-90, PointNameB=P_{13}]{PP_6}{N_6}{O}{P_1}{P_6}{P_13}
  \pstSegmentMark[SegmentSymbol=wedge]{N_6}{P_6}
  \pstSegmentMark[SegmentSymbol=wedge]{P_13}{N_6}
  \pstRotation[RotAngle=90,PointSymbol=none, PointName=none]{N_4}{E}[PP_4]
  \pstInterLC[PosAngleA=90,PosAngleB=-90,PointNameB=P_{15}]{N_4}{PP_4}{O}{P_1}{P_4}{P_15}
  \pstSegmentMark[SegmentSymbol=cup]{N_4}{P_4}
  \pstSegmentMark[SegmentSymbol=cup]{P_15}{N_4}
```

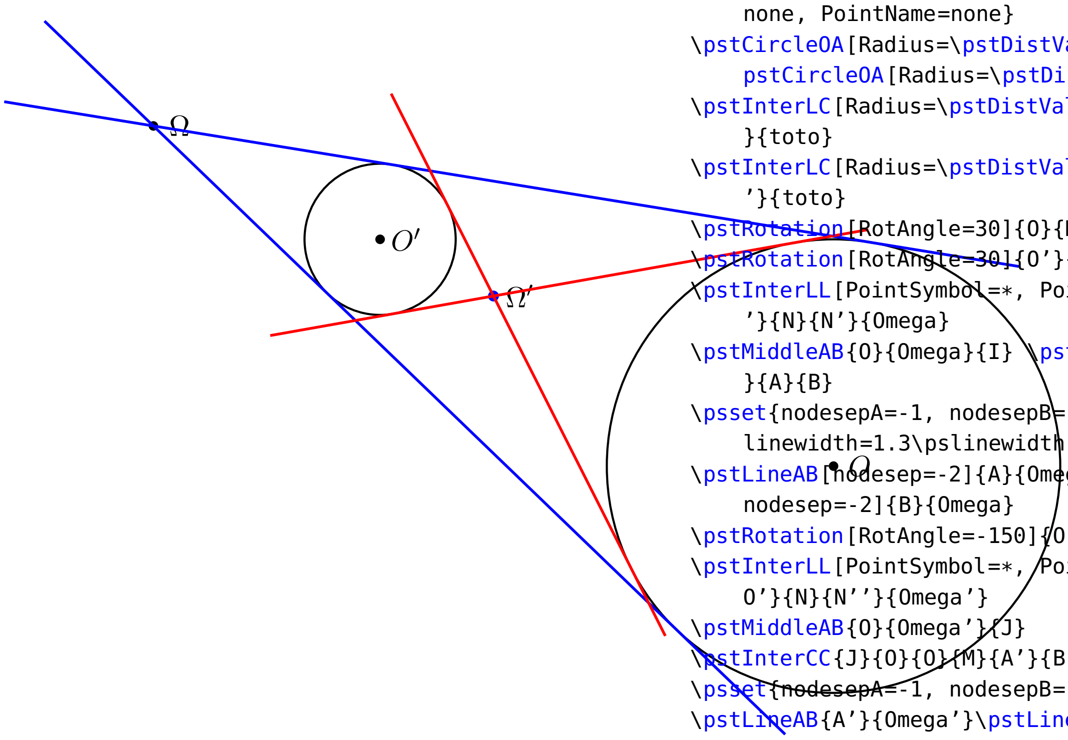
```
  \pstRightAngle[linestyle=solid]{P_1}{N_6}{P_6}
  \pstRightAngle[linestyle=solid]{P_1}{N_4}{P_4}
  \pstBissectBAC[PosAngle=90, linestyle=none]{P_4}{O}{P_6}{P_5}
  \pstInterCC[PosAngleB=90, PointSymbolA=none, PointNameA=none]{O}{P_1}{P_4}{P_5}{H}{P_3}
  \pstInterCC[PosAngleB=90, PointSymbolA=none, PointNameA=none]{O}{P_1}{P_3}{P_4}{H}{P_2}
  \pstInterCC[PosAngleA=90, PointSymbolB=none, PointNameB=none]{O}{P_1}{P_6}{P_5}{P_7}{H}
  \pstInterCC[PosAngleA=100, PointSymbolB=none, PointNameB=none]{O}{P_1}{P_7}{P_6}{P_8}{H}
  \pstInterCC[PosAngleA=135, PointSymbolB=none, PointNameB=none]{O}{P_1}{P_8}{P_7}{P_9}{H}
  \pstOrtSym[PosAngle={-90,-90,-90,-100,-135},PointName={P_{17},P_{16},P_{14},P_{12},P_{11},P_{10}}]
          {O}{P_1}{P_2,P_3,P_5,P_7,P_8,P_9}[P_17,P_16,P_14,P_12,P_11,P_10]
  \pspolygon[linecolor=green, linestyle=solid, linewidth=2\pslinewidth]
    (P_1)(P_2)(P_3)(P_4)(P_5)(P_6)(P_7)(P_8)(P_9)(P_10)(P_11)(P_12)(P_13)(P_14)(P_15)(P_16)(P_17)
\end{pspicture}
```

A.7. Circles & tangents

The drawing of the circle tangents which crosses a given point.



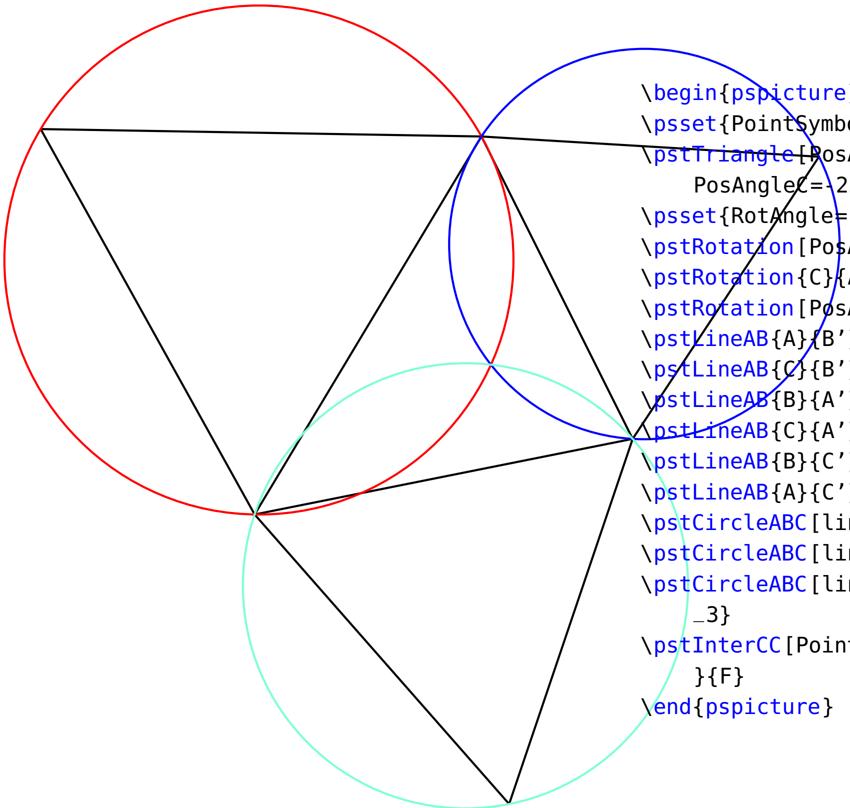

```
\begin{pspicture}(15,10)
\pstGeonode(5, 5){O}(14,2){M}
\pstCircleOA[Radius=\pstDistVal{4}]{O}{}
\pstMiddleAB[PointSymbol=none, PointName=none]{O
    }{M}{O'}
\pstInterCC[RadiusA=\pstDistVal{4}, DiameterB=\
    pstDistAB{O}{M},
         CodeFigB=true, CodeFigColor=magenta,
             PosAngleB=45]{O}{}{O'}{}{A}{B}
\psset{linecolor=red, linewidth=1.3\pslinewidth,
    nodesep=-2}
\pstLineAB{M}{A}\pstLineAB{M}{B}
\end{pspicture}
```




```
\begin{pspicture}(-2,0)(13,9)
\pstGeonode(9,3){O}(3,6){O'}\psset{PointSymbol=
    none, PointName=none}
\pstCircleOA[Radius=\pstDistVal{3}]{O}{}\
    pstCircleOA[Radius=\pstDistVal{1}]{O'}{}
\pstInterLC[Radius=\pstDistVal{3}]{O}{O'}{O}{}{M
    }{toto}
\pstInterLC[Radius=\pstDistVal{1}]{O}{O'}{O'}{}{M
    '}{toto}
\pstRotation[RotAngle=30]{O}{M}[N]
\pstRotation[RotAngle=30]{O'}{M'}[N']
\pstInterLL[PointSymbol=*, PointName=\Omega]{O}{O
    '}{N}{N'}{Omega}
\pstMiddleAB{O}{Omega}{I} \pstInterCC{I}{O}{O}{M
    }{A}{B}
\psset{nodesepA=-1, nodesepB=-3, linecolor=blue,
    linewidth=1.3\pslinewidth}
\pstLineAB[nodesep=-2]{A}{Omega}\pstLineAB[
    nodesep=-2]{B}{Omega}
\pstRotation[RotAngle=-150]{O'}{M'}[N'']
\pstInterLL[PointSymbol=*, PointName=\Omega']{O}{
    O'}{N}{N''}{Omega'}
\pstMiddleAB{O}{Omega'}{J}
\pstInterCC{J}{O}{O}{M}{A'}{B'}
\psset{nodesepA=-1, nodesepB=-3, linecolor=red}
\pstLineAB{A'}{Omega'}\pstLineAB{B'}{Omega'}
\end{pspicture}
```

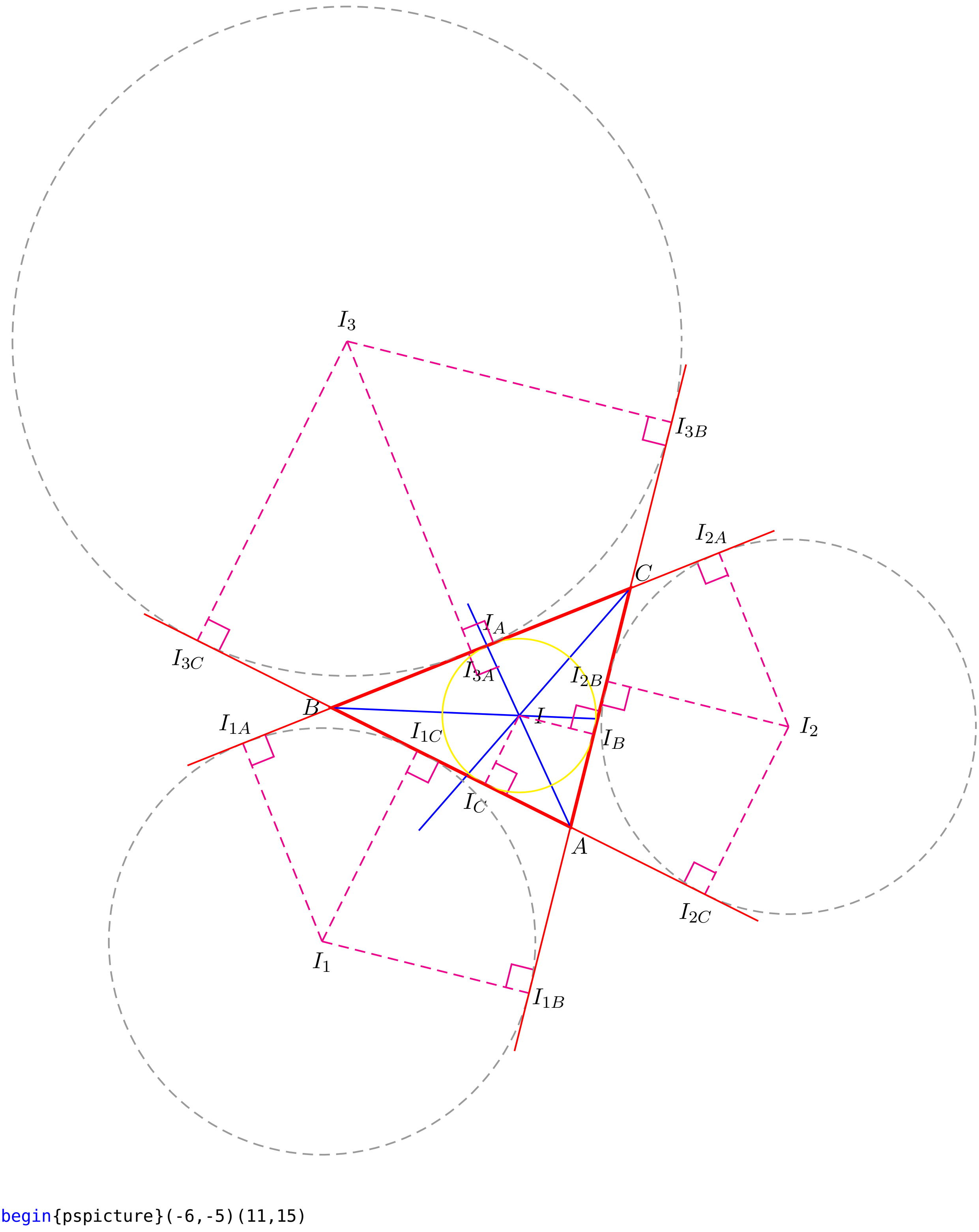
A.8. Fermat's point

Drawing of Manuel Luque.



```
\begin{pspicture}(-7,-6)(5,5)
\psset{PointSymbol=none, PointName=none}
\pstTriangle[PosAngleA=-160,PosAngleB=90,
    PosAngleC=-25](-3,-2){B}(0,3){A}(2,-1){C}%
\psset{RotAngle=-60}
\pstRotation[PosAngle=-90]{B}{C}[A']
\pstRotation{C}{A}[B']
\pstRotation[PosAngle=160]{A}{B}[C']
\pstLineAB{A}{B'}
\pstLineAB{C}{B'}
\pstLineAB{B}{A'}
\pstLineAB{C}{A'}
\pstLineAB{B}{C'}
\pstLineAB{A}{C'}
\pstCircleABC[linecolor=red]{A}{B}{C'}{O_1}
\pstCircleABC[linecolor=blue]{A}{C}{B'}{O_2}
\pstCircleABC[linecolor=Aquamarine]{A'}{C}{B}{O
    _3}
\pstInterCC[PointSymbolA=none]{O_1}{A}{O_2}{A}{E
    }{F}
\end{pspicture}
```

A.9. Escribed and inscribed circles of a triangle



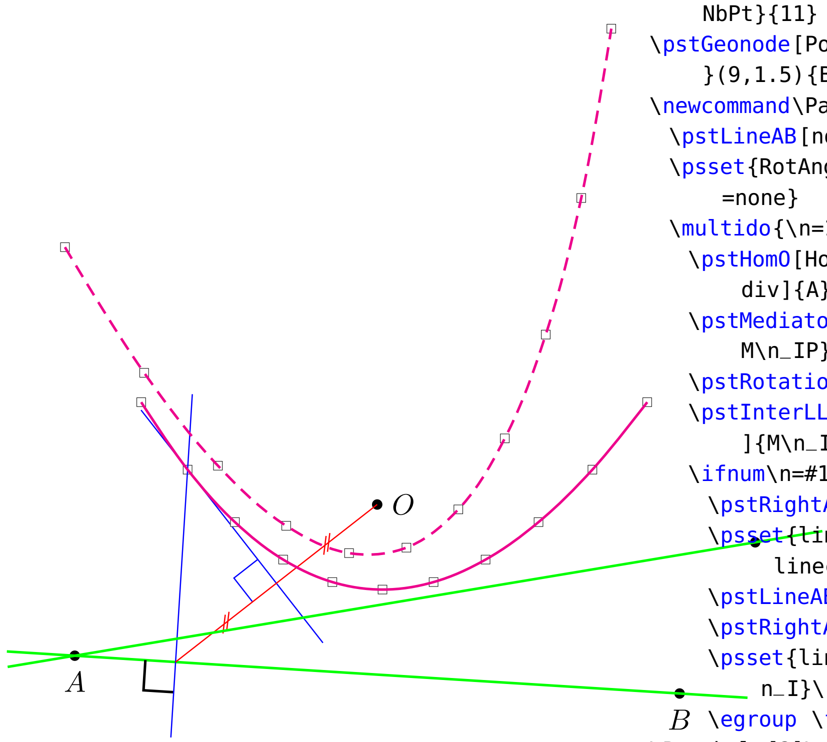

```
\begin{pspicture}(-6,-5)(11,15)
\psset{PointSymbol=none}
\pstTriangle[linewidth=2\pslinewidth,linecolor=red](4,1){A}(0,3){B}(5,5){C}
\psset{linecolor=blue}
\pstBissectBAC[PointSymbol=none,PointName=none]{C}{A}{B}{AB}
\pstBissectBAC[PointSymbol=none,PointName=none]{A}{B}{C}{BB}
\pstBissectBAC[PointSymbol=none,PointName=none]{B}{C}{A}{CB}
\pstInterLL{A}{AB}{B}{BB}{I}
```

```
\psset{linecolor=magenta, linestyle=dashed}
\pstProjection{A}{B}{I}[I_C]
\pstLineAB{I}{I_C}\pstRightAngle[linestyle=solid]{A}{I_C}{I}
\pstProjection{A}{C}{I}[I_B]
\pstLineAB{I}{I_B}\pstRightAngle[linestyle=solid]{C}{I_B}{I}
\pstProjection[PosAngle=80]{C}{B}{I}[I_A]
\pstLineAB{I}{IA}\pstRightAngle[linestyle=solid]{B}{I_A}{I}
\pstCircleOA[linecolor=yellow, linestyle=solid]{I}{I_A}
\psset{linecolor=magenta, linestyle=none}
\pstOutBissectBAC[PointSymbol=none,PointName=none]{C}{A}{B}{AOB}
\pstOutBissectBAC[PointSymbol=none,PointName=none]{A}{B}{C}{BOB}
\pstOutBissectBAC[PointSymbol=none,PointName=none]{B}{C}{A}{COB}
\pstInterLL[PosAngle=-90]{A}{AOB}{B}{BOB}{I_1}
\pstInterLL{A}{AOB}{C}{COB}{I_2}
\pstInterLL[PosAngle=90]{C}{COB}{B}{BOB}{I_3}
\psset{linecolor=magenta, linestyle=dashed}
\pstProjection[PointName=I_{1C}]{A}{B}{I_1}[I1C]
\pstLineAB{I_1}{I1C}\pstRightAngle[linestyle=solid]{I_1}{I1C}{A}
\pstProjection[PointName=I_{1B}]{A}{C}{I_1}[I1B]
\pstLineAB{I_1}{I1B}\pstRightAngle[linestyle=solid]{A}{I1B}{I_1}
\pstProjection[PointName=I_{1A}]{C}{B}{I_1}[I1A]
\pstLineAB{I_1}{I1A}\pstRightAngle[linestyle=solid]{I_1}{I1A}{C}
\pstProjection[PointName=I_{2B}]{A}{C}{I_2}[I2B]
\pstLineAB{I_2}{I2B}\pstRightAngle[linestyle=solid]{A}{I2B}{I_2}
\pstProjection[PointName=I_{2C}]{A}{B}{I_2}[I2C]
\pstLineAB{I_2}{I2C}\pstRightAngle[linestyle=solid]{I_2}{I2C}{A}
\pstProjection[PointName=I_{2A}]{B}{C}{I_2}[I2A]
\pstLineAB{I_2}{I2A}\pstRightAngle[linestyle=solid]{C}{I2A}{I_2}
\pstProjection[PointName=I_{3A}]{C}{B}{I_3}[I3A]
\pstLineAB{I_3}{I3A}\pstRightAngle[linestyle=solid]{C}{I3A}{I_3}
\pstProjection[PointName=I_{3C}]{A}{B}{I_3}[I3C]
\pstLineAB{I_3}{I3C}\pstRightAngle[linestyle=solid]{A}{I3C}{I_3}
\pstProjection[PointName=I_{3B}]{C}{A}{I_3}[I3B]
\pstLineAB{I_3}{I3B}\pstRightAngle[linestyle=solid]{I_3}{I3B}{A}
\psset{linecolor=yellow, linestyle=solid}
\pstCircleOA{I_1}{I1C} \pstCircleOA{I_2}{I2B} \pstCircleOA{I_3}{I3A}
\psset{linecolor=red, linestyle=solid, nodesepA=-1, nodesepB=-1}
\pstLineAB{I1B}{I3B}\pstLineAB{I1A}{I2A}\pstLineAB{I2C}{I3C}
\end{pspicture}
```

B. Some locus points

B.1. Parabola

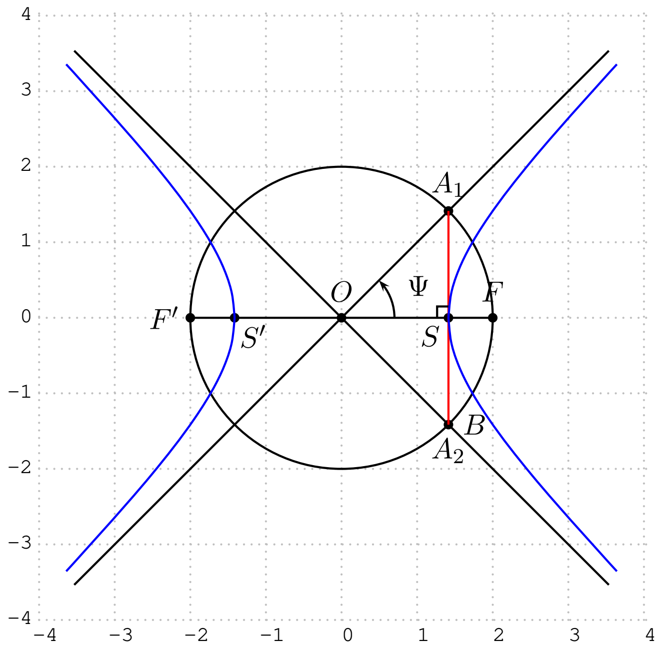
The parabola is the set of points which are at the same distance between a point and a line.




```
\def\NbPt{11}
\begin{pspicture}(-0.5,0)(11,10)
\psset{linewidth=1.2\pslinewidth}\renewcommand{\
    NbPt}{11}
\pstGeonode[PosAngle={0,-90}](5,4){O}(1,2){A
    }(9,1.5){B}
\newcommand\Parabole[1][100]{%
  \pstLineAB[nodesep=-.9, linecolor=green]{A}{B}
  \psset{RotAngle=90, PointSymbol=none, PointName
      =none}
  \multido{\n=1+1}{\NbPt}{%
    \pstHomO[HomCoef=\n\space \NbPt\space 1 add
        div]{A}{B}[M\n]
    \pstMediatorAB[linestyle=none]{M\n}{O}{M\n_I}{
        M\n_IP}
    \pstRotation{M\n}{A}[M\n_P]
    \pstInterLL[PointSymbol=square, PointName=none
        ]{M\n_I}{M\n_IP}{M\n}{M\n_P}{P_\n}
    \ifnum\n=#1 \bgroup
      \pstRightAngle{A}{M\n}{M\n_P}
      \psset{linewidth=.5\pslinewidth, nodesep=-1,
          linecolor=blue}
      \pstLineAB{M\n_I}{P_\n}\pstLineAB{M\n}{P_\n}
      \pstRightAngle{P_\n}{M\n_I}{M\n}
      \psset{linecolor=red}\pstSegmentMark{M\n}{M\
          n_I}\pstSegmentMark{M\n_I}{O}
      \egroup \fi}}
\Parabole[2]\pstGenericCurve[linecolor=magenta]{P
    _}{1}{\NbPt}
\pstGeonode[PointSymbol=*, PosAngle=-90](10,3.5){
    B}
\Parabole\pstGenericCurve[linecolor=magenta,
    linestyle=dashed]{P_}{1}{\NbPt}
\end{pspicture}
```

B.2. Hyperbola

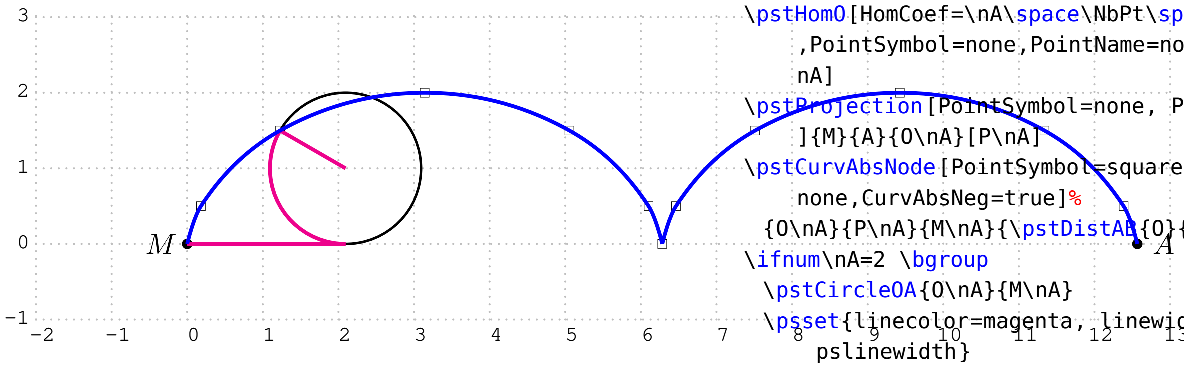
The hyperbola is the set of points whose difference between their distance of two points (the focus) is constant.




```
\begin{pspicture}[showgrid](-4,-4)(4,4)
\newcommand\Sommet{1.4142135623730951 } \
    newcounter{i} \setcounter{i}{1}
\newcommand\PosFoyer{2 } \newcommand\HypAngle{0}
\newcounter{CoefDiv}\setcounter{CoefDiv}{20}
\newcounter{Inc}\setcounter{Inc}{2} \newcounter{n
    }\setcounter{n}{2}
\newcommand\Ri{ \PosFoyer \Sommet sub \arabic{i}\
    space\arabic{CoefDiv}\space div add }
\newcommand\Rii{\Ri \Sommet 2 mul add }
\pstGeonode[PosAngle=90]{O}(\PosFoyer;\HypAngle){
    F}
\pstSymO[PosAngle=180]{O}{F}\pstLineAB{F}{F'} \
    pstCircleOA{O}{F}
\pstGeonode[PosAngle=-135](\Sommet;\HypAngle){S}
\pstGeonode[PosAngle=-45](-\Sommet;\HypAngle){S'}
\pstRotation[RotAngle=90, PointSymbol=none]{S}{O
    }[B]
\pstInterLC[PosAngleA=90, PosAngleB=-90]{S}{B}{O
    }{F}{A_1}{A_2}
\pstLineAB[nodesepA=-3,nodesepB=-5]{A_1}{O}\
    pstLineAB[nodesepA=-3,nodesepB=-5]{A_2}{O}
\pstMarkAngle[LabelSep=.8,MarkAngleRadius=.7,
    arrows=->,LabelSep=1.1]{F}{O}{A_1}{$\Psi$}
\ncline[linecolor=red]{A_1}{A_2} \pstRightAngle[
    RightAngleSize=.15]{A_1}{S}{O}
\psset{PointName=none}
\whiledo{\value{n}<8}{%
  \psset{RadiusA=\pstDistVal{\Ri},RadiusB=\
      pstDistVal{\Rii},PointSymbol=none}
  \pstInterCC{F}{}{F'}{}{M\arabic{n}}{P\arabic{n
      }}
  \pstInterCC{F'}{}{F}{}{M'\arabic{n}}{P'\arabic{
      n}}
  \stepcounter{n}\addtocounter{i}{\value{Inc}}
  \addtocounter{Inc}{\value{Inc}}}%% fin de
      whiledo
\psset{linecolor=blue}
\pstGenericCurve[GenCurvFirst=S]{M}{2}{7}
\pstGenericCurve[GenCurvFirst=S]{P}{2}{7}
\pstGenericCurve[GenCurvFirst=S']{M'}{2}{7}
\pstGenericCurve[GenCurvFirst=S']{P'}{2}{7}
\end{pspicture}
```

B.3. Cycloid

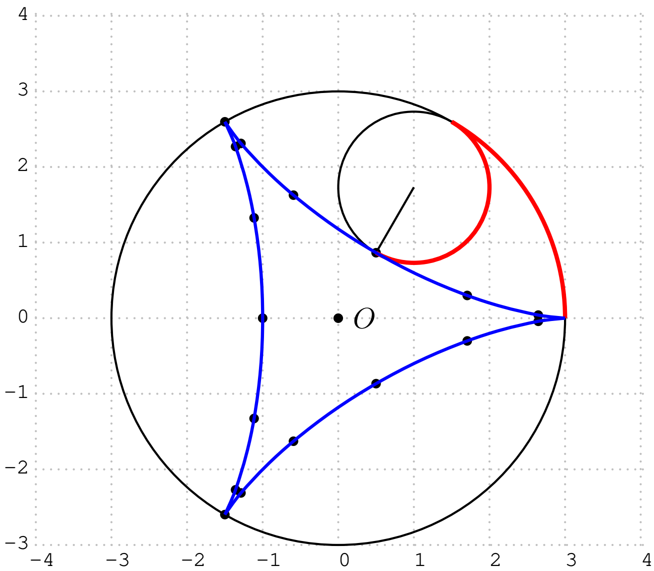
The wheel rolls from M to A. The circle points are on a cycloid.



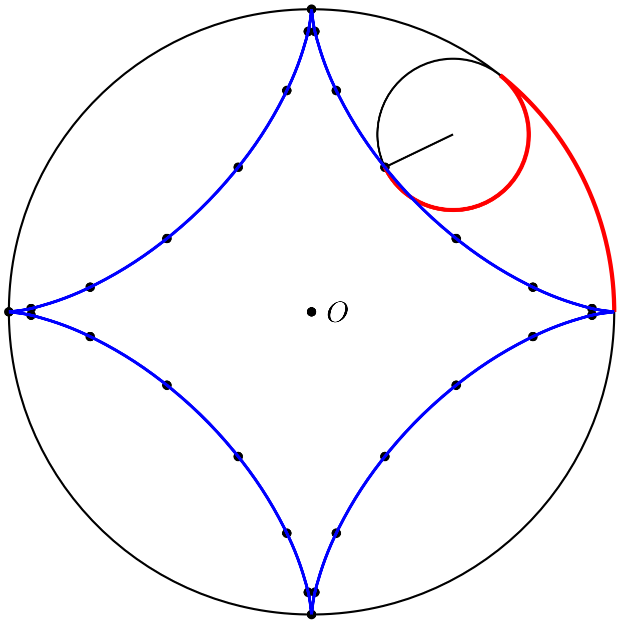

```
\begin{pspicture}[showgrid](-2,-1)(13,3)
\providecommand\NbPt{11}
\psset{linewidth=1.2\pslinewidth}
\pstGeonode[PointSymbol={*,none}, PointName={
    default,none}, PosAngle=180]{M}(0,1){O}
\pstGeonode(12.5663706144,0){A}
\pstTranslation[PointSymbol=none, PointName=none
    ]{M}{A}{O}[B]
\multido{\nA=1+1}{\NbPt}{%
  \pstHomO[HomCoef=\nA\space\NbPt\space 1 add div
      ,PointSymbol=none,PointName=none]{O}{B}[O\
      nA]
  \pstProjection[PointSymbol=none, PointName=none
      ]{M}{A}{O\nA}[P\nA]
  \pstCurvAbsNode[PointSymbol=square, PointName=
      none,CurvAbsNeg=true]%
    {O\nA}{P\nA}{M\nA}{\pstDistAB{O}{O\nA}}
  \ifnum\nA=2 \bgroup
    \pstCircleOA{O\nA}{M\nA}
    \psset{linecolor=magenta, linewidth=1.5\
        pslinewidth}
    \pstArcnOAB{O\nA}{P\nA}{M\nA}
    \ncline{O\nA}{M\nA}\ncline{P\nA}{M}
    \egroup \fi
  }% fin du multido
\psset{linecolor=blue, linewidth=1.5\pslinewidth}
\pstGenericCurve[GenCurvFirst=M]{M}{1}{6} \
    pstGenericCurve[GenCurvLast=A]{M}{6}{\NbPt}
\end{pspicture}
```

B.4. Hypocycloids (Astroid and Deltoid)

A wheel rolls inside a circle, and depending of the radius ratio, it is an astroid, a deltoid and in the general case hypo-cycloids.




```
\newcommand\HypoCyclo[4][100]{%
  \def\R{#2}\def\petitR{#3}\def\NbPt{#4}
  \def\Anglen{\n\space 360 \NbPt\space 1 add div
      mul}
  \psset{PointSymbol=none,PointName=none}
  \pstGeonode[PointSymbol={*,none},PointName={
      default,none}, PosAngle=0]{O}(\R;0){P}
  \pstCircleOA{O}{P}
  \pstHomO[HomCoef=\petitR\space\R\space div]{P}{
      O}[M]
  \multido{\n=1+1}{\NbPt}{%
    \pstRotation[RotAngle=\Anglen]{O}{M}[M\n]
    \rput(M\n){\pstGeonode(\petitR;0){Q}}
    \pstRotation[RotAngle=\Anglen]{M\n}{Q}[N]
    \pstRotation[RotAngle=\n\space -360 \NbPt\
        space 1 add div
    mul \R\space\petitR\space div mul,PointSymbol
        =*,PointName=none]{M\n}{N}[N\n]
    \ifnum\n=#1
      \pstCircleOA{M\n}{N\n}\ncline{M\n}{N\n}%
      {\psset{linecolor=red, linewidth=2\
          pslinewidth}
      \pstArcOAB{M\n}{N\n}{N}\pstArcOAB{O}{P}{N}}
    \fi}}%fin multido-newcommand
\begin{pspicture}[showgrid](-3.5,-3.4)(3.5,4)
\HypoCyclo[3]{3}{1}{17}
\psset{linecolor=blue,linewidth=1.5\pslinewidth}
\pstGenericCurve[GenCurvFirst=P]{N}{1}{6}
\pstGenericCurve{N}{6}{12}
\pstGenericCurve[GenCurvLast=P]{N}{12}{17}
\end{pspicture}
```




```
\newcommand\HypoCyclo[4][100]{%
  \def\R{#2}\def\petitR{#3}\def\NbPt{#4}
  \def\Anglen{\n\space 360 \NbPt\space 1 add div
      mul}
  \psset{PointSymbol=none,PointName=none}
  \pstGeonode[PointSymbol={*,none},PointName={
      default,none}, PosAngle=0]{O}(\R;0){P}
  \pstCircleOA{O}{P}
  \pstHomO[HomCoef=\petitR\space\R\space div]{P}{
      O}[M]
  \multido{\n=1+1}{\NbPt}{%
    \pstRotation[RotAngle=\Anglen]{O}{M}[M\n]
    \rput(M\n){\pstGeonode(\petitR;0){Q}}
    \pstRotation[RotAngle=\Anglen]{M\n}{Q}[N]
    \pstRotation[RotAngle=\n\space -360 \NbPt\
        space 1 add div
    mul \R\space\petitR\space div mul, PointSymbol
        =*, PointName=none]{M\n}{N}[N\n]
    \ifnum\n=#1
      \pstCircleOA{M\n}{N\n}\ncline{M\n}{N\n}%
      {\psset{linecolor=red, linewidth=2\
          pslinewidth}
      \pstArcOAB{M\n}{N\n}{N}\pstArcOAB{O}{P}{N}}
    \fi}}%fin multido-newcommand
\begin{pspicture}(-4.5,-4)(4.5,4.5)
\HypoCyclo[4]{4}{1}{27}
\psset{linecolor=blue, linewidth=1.5\pslinewidth}
\pstGenericCurve[GenCurvFirst=P]{N}{1}{7}
\pstGenericCurve{N}{7}{14}\pstGenericCurve{N
    }{14}{21}
\pstGenericCurve[GenCurvLast=P]{N}{21}{27}
\end{pspicture}
```

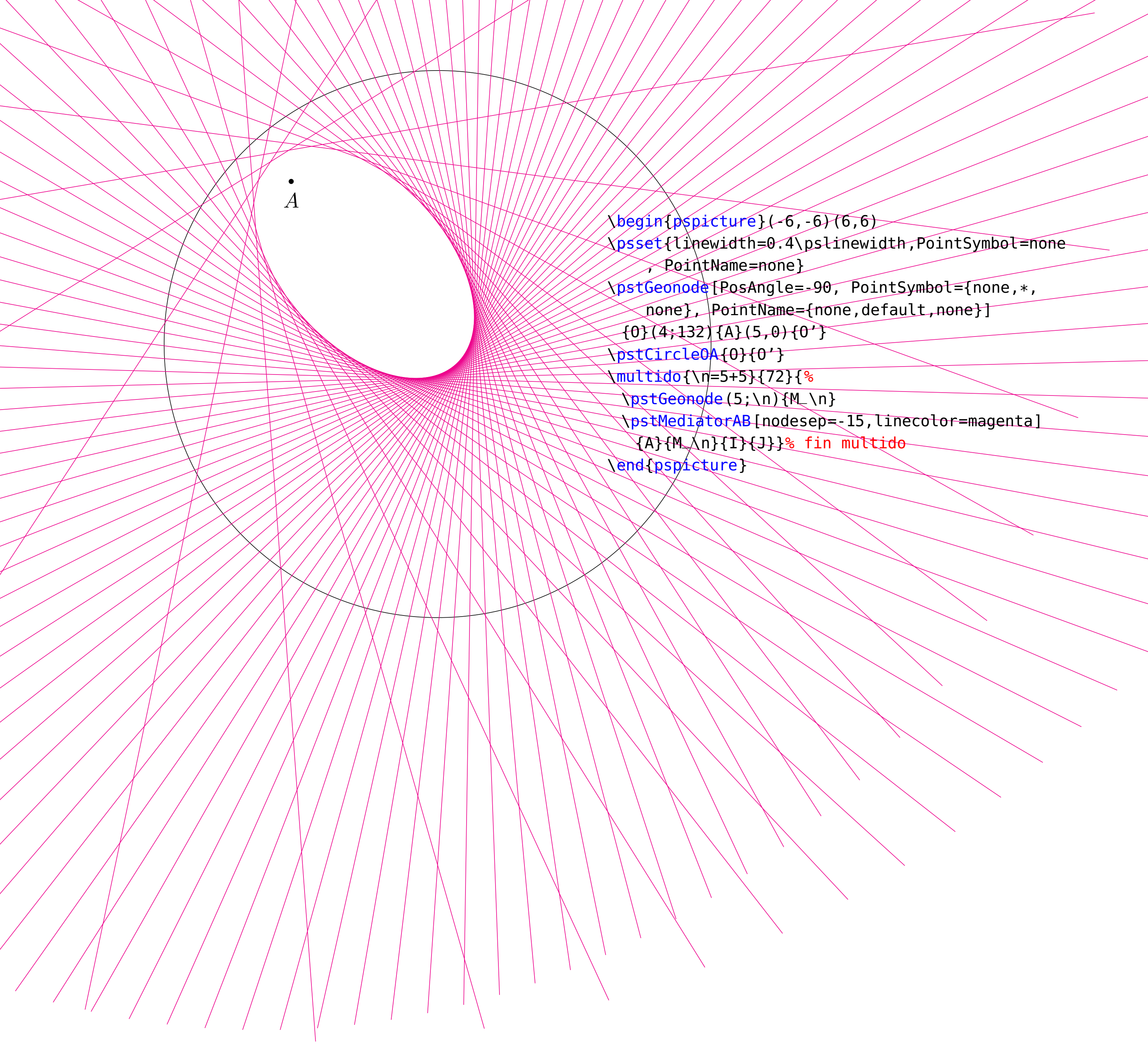
C. Lines and circles envelope

C.1. Conics

Let's consider a circle and a point A not on the circle. The set of all the mediator lines of segments defined by A and the circle points, create two conics depending of the position of A:

- inside the circle: a hyperbola;
- outside the circle: an ellipse.

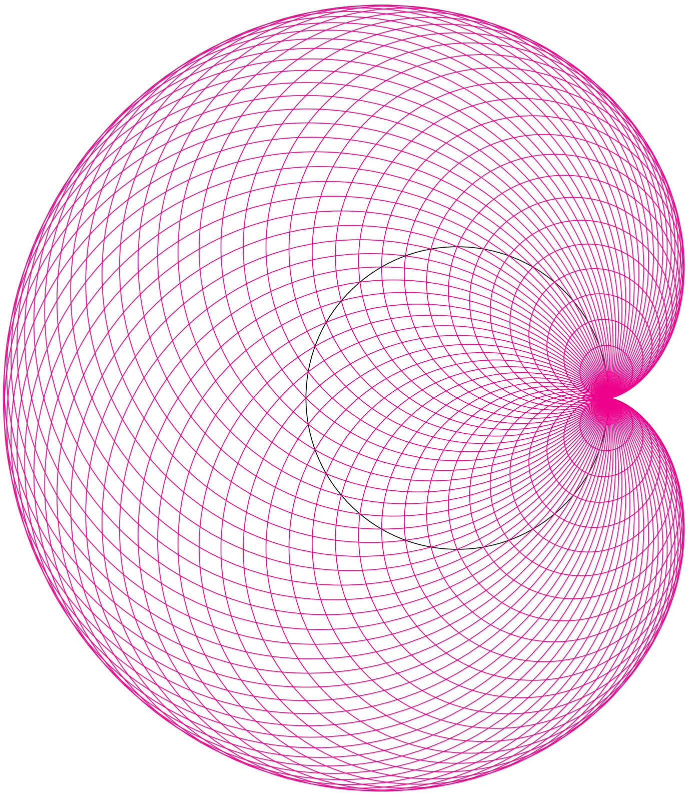
(figure of O. Reboux).




```
\begin{pspicture}(-6,-6)(6,6)
\psset{linewidth=0.4\pslinewidth,PointSymbol=none
    , PointName=none}
\pstGeonode[PosAngle=-90, PointSymbol={none,*,
    none}, PointName={none,default,none}]
  {O}(4;132){A}(5,0){O'}
\pstCircleOA{O}{O'}
\multido{\n=5+5}{72}{%
  \pstGeonode(5;\n){M_\n}
  \pstMediatorAB[nodesep=-15,linecolor=magenta]
   {A}{M_\n}{I}{J}}% fin multido
\end{pspicture}
```

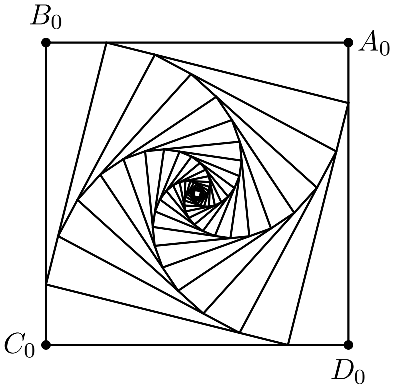
C.2. Cardioid

The cardioid is defined by the circles centered on a circle and crossing a given point.



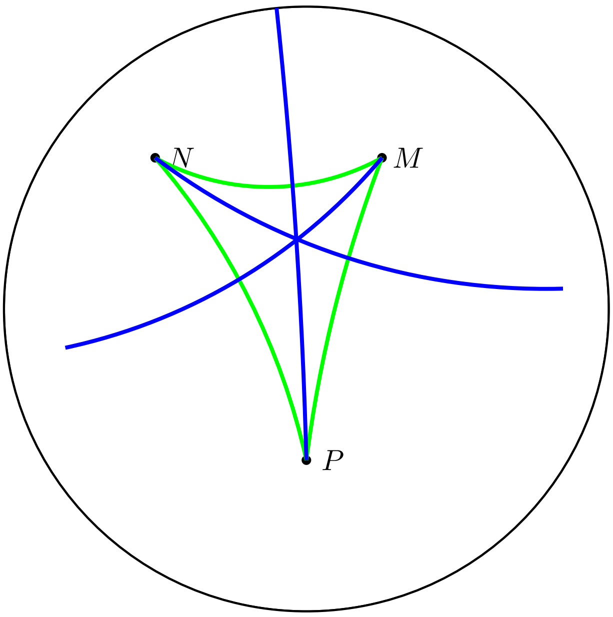
```
\begin{pspicture}(-6,-6)(3,5)
\psset{linewidth=0.4\pslinewidth,PointSymbol=x,
    nodesep=0,linecolor=magenta}
\pstGeonode[PointName=none]{O}(2,0){O'}
\pstCircleOA[linecolor=black]{O}{O'}
\multido{\n=5+5}{72}{%
  \pstGeonode[PointSymbol=none, PointName=none
     ](2;\n){M_\n}
  \pstCircleOA{M_\n}{O'}}
  \end{pspicture}
```

D. Homotethy and fractals




```
\begin{pspicture}(-2.8,-3)(2.8,3)
\pstGeonode[PosAngle={0,90}](2,2){A_0}(-2,2){B_0}%
\psset{RotAngle=90}
\pstRotation[PosAngle=270]{A_0}{B_0}[D_0]
\pstRotation[PosAngle=180]{D_0}{A_0}[C_0]
\pspolygon(A_0)(B_0)(C_0)(D_0)%
\psset{PointSymbol=none, PointName=none, HomCoef=.2}
\multido{\n=1+1,\i=0+1}{20}{%
 \pstHomO[PosAngle=0]{B_\i}{A_\i}[A_\n]
 \pstHomO[PosAngle=90]{C_\i}{B_\i}[B_\n]
 \pstHomO[PosAngle=180]{D_\i}{C_\i}[C_\n]
 \pstHomO[PosAngle=270]{A_\i}{D_\i}[D_\n]
 \pspolygon(A_\n)(B_\n)(C_\n)(D_\n)}% fin multido
\end{pspicture}
```

E. hyperbolic geometry: a triangle and its altitudes




```
\begin{pspicture}(-5,-5)(5,5)
\psclip{\pscircle(0,0){4}}
  \pstGeonode(1, 2){M}\pstGeonode(-2,2){N}\
      pstGeonode(0,-2){P}
  \psset{DrawCirABC=false, PointSymbol=none,
      PointName=none}%
  \pstGeonode(0,0){O}\pstGeonode(4,0){A}\
      pstCircleOA{O}{A}
  \pstHomO[HomCoef=\pstDistAB{O}{A} 2 mul \
      pstDistAB{O}{M} sub
   \pstDistAB{O}{M} div]{O}{M}[M']%
  \pstHomO[HomCoef=\pstDistAB{O}{A} 2 mul \
      pstDistAB{O}{P} sub
   \pstDistAB{O}{P} div]{O}{P}[P']%
  \pstHomO[HomCoef=\pstDistAB{O}{A} 2 mul \
      pstDistAB{O}{N} sub
   \pstDistAB{O}{N} div]{O}{N}[N']%
  \psset{linecolor=green, linewidth=1.5pt}%
  \pstCircleABC{M}{N}{M'}{OmegaMN}\pstArcOAB{
      OmegaMN}{N}{M}
  \pstCircleABC{M}{P}{M'}{OmegaMP}\pstArcOAB{
      OmegaMP}{M}{P}
  \pstCircleABC{N}{P}{P'}{OmegaNP}\pstArcOAB{
      OmegaNP}{P}{N}
  \psset{linecolor=blue}
  \pstHomO[HomCoef=\pstDistAB{OmegaNP}{N} 2 mul \
      pstDistAB{OmegaNP}{M} sub %% M
   \pstDistAB{OmegaNP}{M} div]{OmegaNP}{M}[MH']
  \pstCircleABC{M}{M'}{MH'}{OmegaMH}\pstArcOAB{
      OmegaMH}{MH'}{M} %% N
  \pstHomO[HomCoef=\pstDistAB{OmegaMP}{M} 2 mul \
      pstDistAB{OmegaMP}{N} sub
   \pstDistAB{OmegaMP}{N} div]{OmegaMP}{N}[NH']
  \pstCircleABC{N}{N'}{NH'}{OmegaNH}\pstArcOAB{
      OmegaNH}{N}{NH'} %% P
  \pstHomO[HomCoef=\pstDistAB{OmegaMN}{M} 2 mul \
      pstDistAB{OmegaMN}{P} sub
   \pstDistAB{OmegaMN}{P} div]{OmegaMN}{P}[PH']
  \pstCircleABC{P}{P'}{PH'}{OmegaPH}\pstArcOAB{
      OmegaPH}{P}{PH'}
\endpsclip
\end{pspicture}
```

F. List of all optional arguments for `pst-eucl`

Key	Type	Default
PointSymbol	ordinary	*
PointSymbolA	ordinary	*
PointSymbolB	ordinary	*
PointSymbolC	ordinary	*
PointName	ordinary	default
PointNameA	ordinary	undef
PointNameB	ordinary	undef
PointNameC	ordinary	undef
PtNameMath	ordinary	false
PointNameSize	ordinary	\normalsize
PointNameMathSize	ordinary	\textnormal
SegmentSymbol	ordinary	MarkHashh
SegmentSymbolA	ordinary	MarkHashh
SegmentSymbolB	ordinary	MarkHashh
SegmentSymbolC	ordinary	MarkHashh
Mark	ordinary	undef
mark	ordinary	undef
MarkAngle	ordinary	undef
MarkHashLength	ordinary	1.25mm
MarkHashSep	ordinary	0.625mm
PointNameSep	ordinary	[none]
PosAngle	ordinary	[none]
PosAngleA	ordinary	undef
PosAngleB	ordinary	undef
PosAngleC	ordinary	undef
RightAngleSize	ordinary	4
RightAngleType	ordinary	default
RightAngleDotDistance	ordinary	1
MarkAngleRadius	ordinary	0.4
MarkAngleType	ordinary	default
LabelAngleOffset	ordinary	0
LabelSep	ordinary	1
LabelRefPt	ordinary	c
CurveType	ordinary	none
HomCoef	ordinary	0.5
RotAngle	ordinary	60
TransformLabel	ordinary	none
Central@Sym	ordinary	false
DrawCirABC	ordinary	true
CodeFig	boolean	true
CodeFigA	ordinary	undef
CodeFigB	ordinary	undef
CodeFigColor	ordinary	cyan
CodeFigStyle	ordinary	dashed

Continued on next page

Continued from previous page

Key	Type	Default
CodeFigAarc	ordinary	true
CodeFigBarc	ordinary	true
Radius	ordinary	none
RadiusA	ordinary	undef
RadiusB	ordinary	undef
Diameter	ordinary	none
DiameterA	ordinary	undef
DiameterB	ordinary	undef
DistCoef	ordinary	none
AngleCoef	ordinary	none
CurvAbsNeg	ordinary	false
GenCurvFirst	ordinary	none
GenCurvLast	ordinary	none
GenCurvInc	ordinary	1
AngleValue	boolean	false
AngleArc	boolean	true
ShowWedge	boolean	true
ArcColor	ordinary	[none]
ArcLinestyle	ordinary	[none]
ArcLinewidth	ordinary	[none]
WedgeColor	ordinary	[none]
WedgeFillstyle	ordinary	[none]
WedgeOpacity	ordinary	[none]

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[11] Timothy Van Zandt and Denis Girou. “Inside PSTricks”. In: *TUGboat* 15 (Sept. 1994), pp. 239–246.

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